Quantum Computing

Dieter Suter
Gonzalo A. Álvarez, Analía Zwick

Laboratory of Nuclear Magnetic Resonance Spectroscopy and Imaging
Balseiro Institute - CONICET

Bariloche, Argentina

CONICET
CNEA
Centro Atómico Bariloche
Instituto Balseiro

Contact:
Gonzalo A. Alvarez: gonzalo.alvarez@conicet.gov.ar
gonzalo.a.alvarez@gmail.com
Analia Zwick: analia.zwick@gmail.com
Web-page: https://www.famaf.unc.edu.ar/~galvarez/

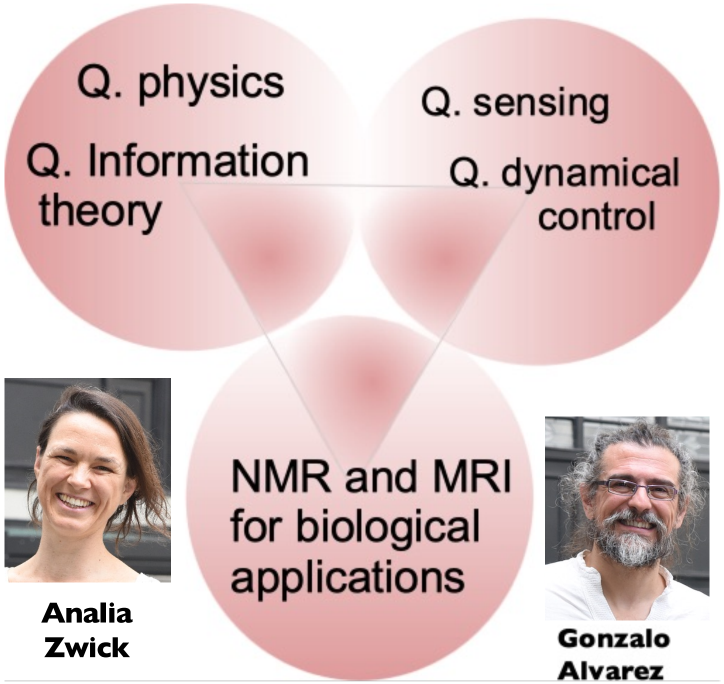
Quantum Computing

Dieter Suter

Gonzalo A. Álvarez, Analía Zwick

Laboratory of Nuclear Magnetic Resonance Spectroscopy and Imaging

Balseiro Institute - CONICET



CONICET

Centro Atómico Bariloche

Instituto Balseiro

Contact:

Gonzalo A. Alvarez: gonzalo.alvarez@conicet.gov.ar

gonzalo.a.alvarez@gmail.com

Analia Zwick: analia.zwick@gmail.com

Web-page: https://www.famaf.unc.edu.ar/~galvarez/

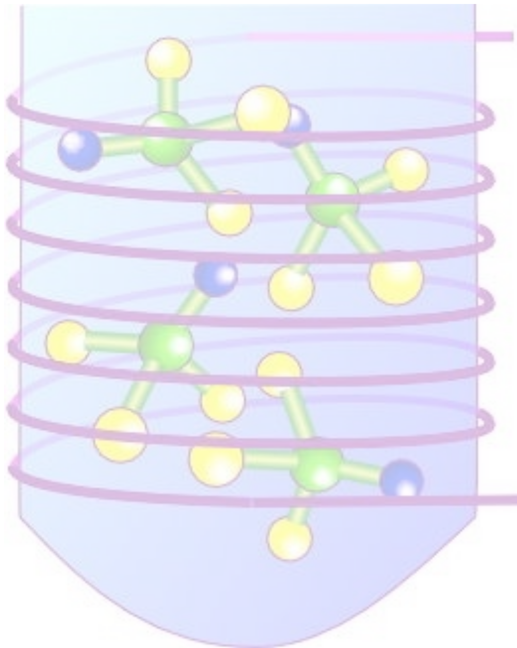
Contents

Nuclear Magnetic Resonance (NMR)

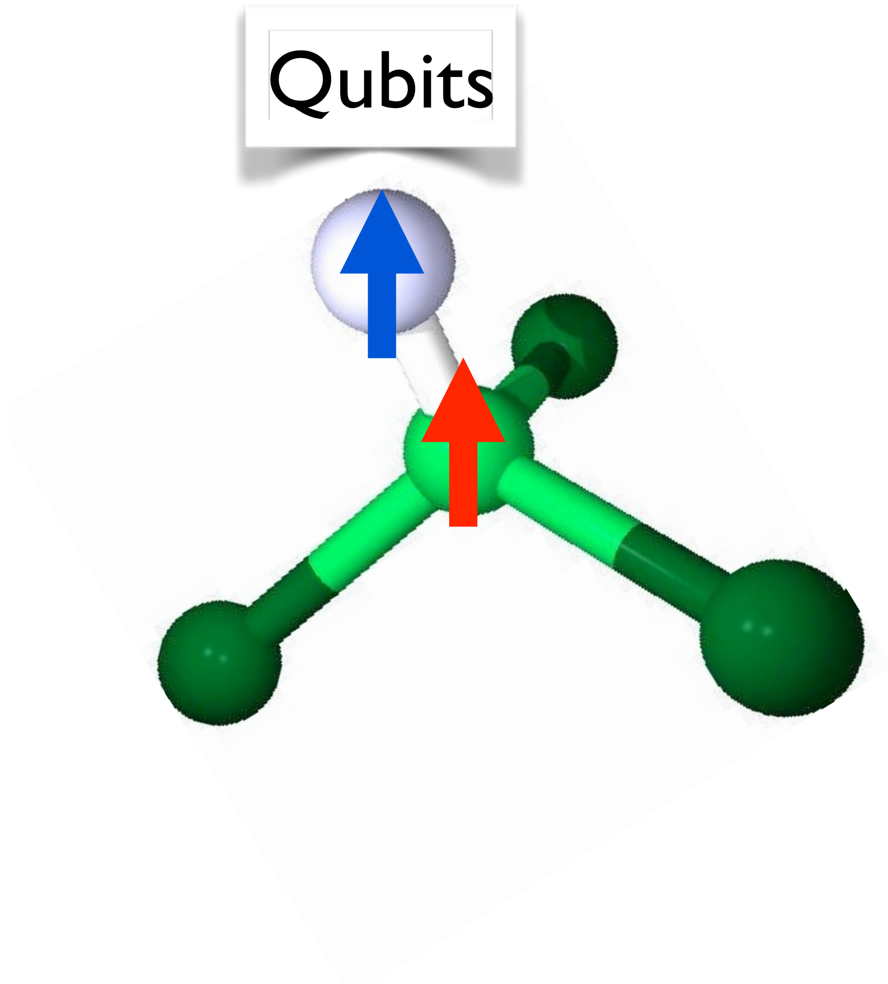
Why Spins?

- Simple, well isolated quantum system
- Long decoherence times (~sec)
- Gate operations are "easy" to implement
- Mature Technology

+ it works!



Ensemble Quantum Computing

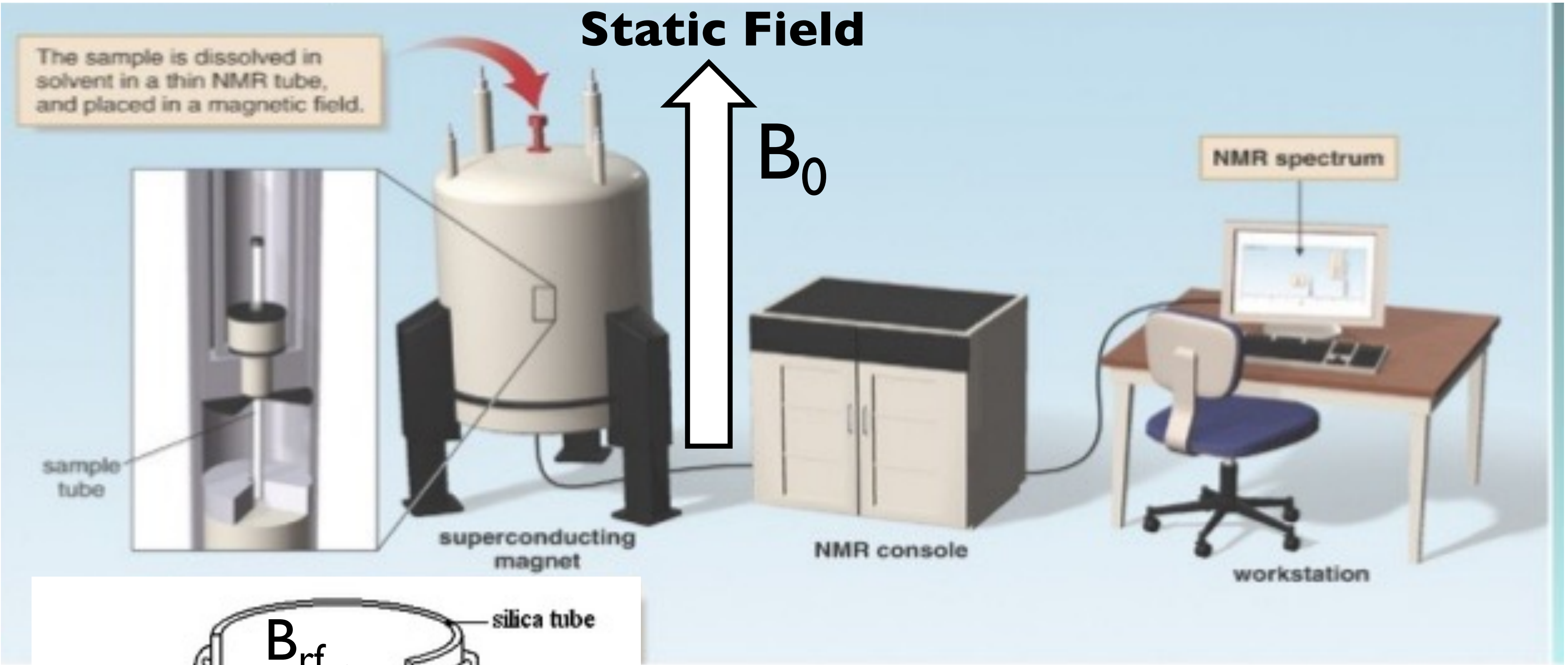


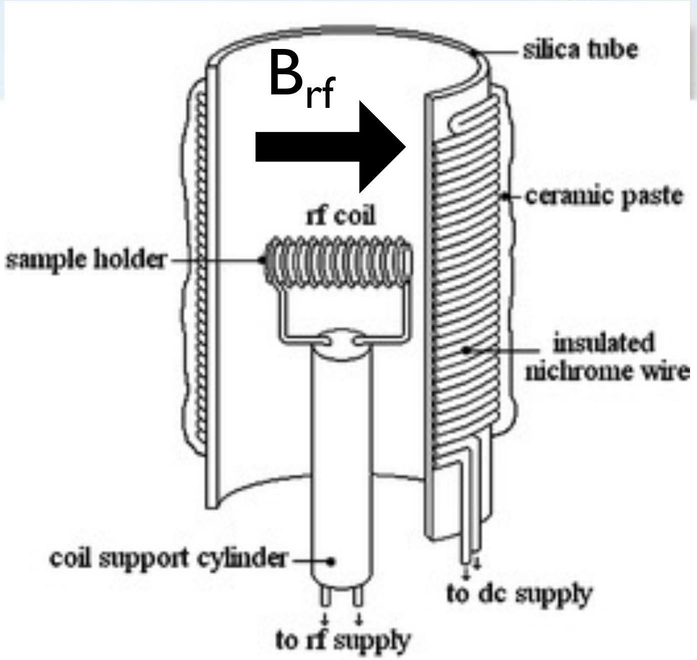
Single spin detection difficult

use 10^{20} identical copies

"Ensemble Quantum Computing"

Liquid-state NMR Equipments

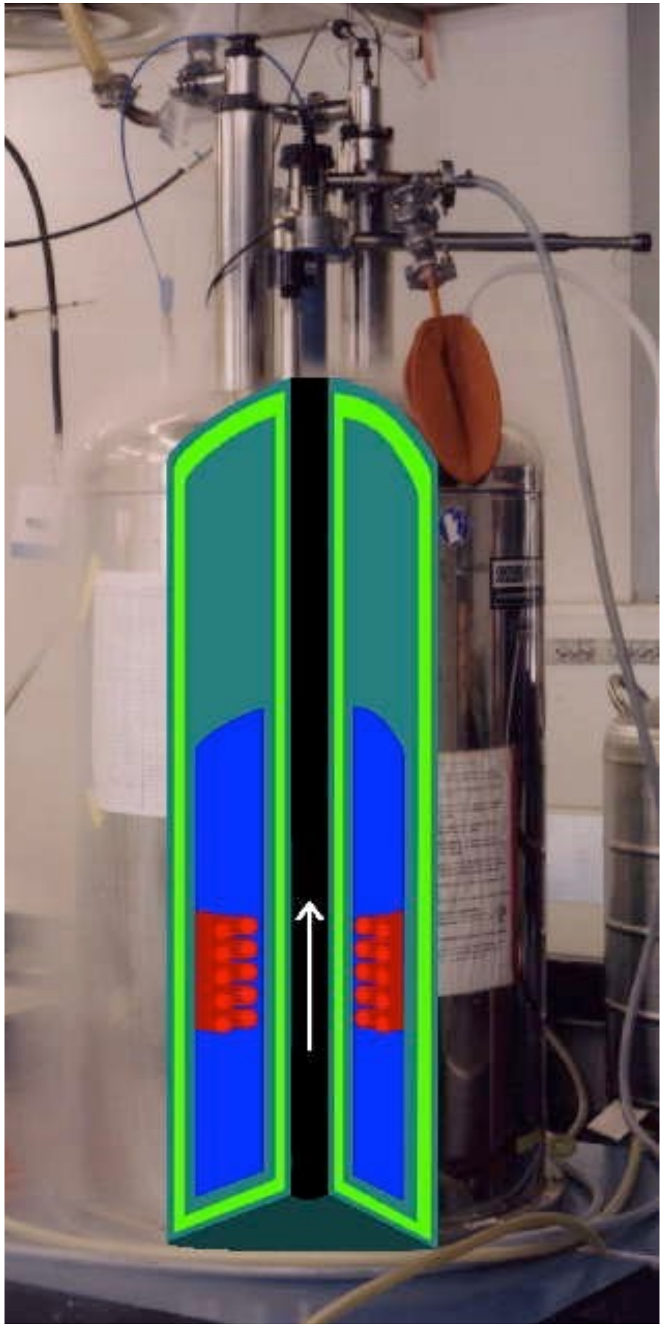




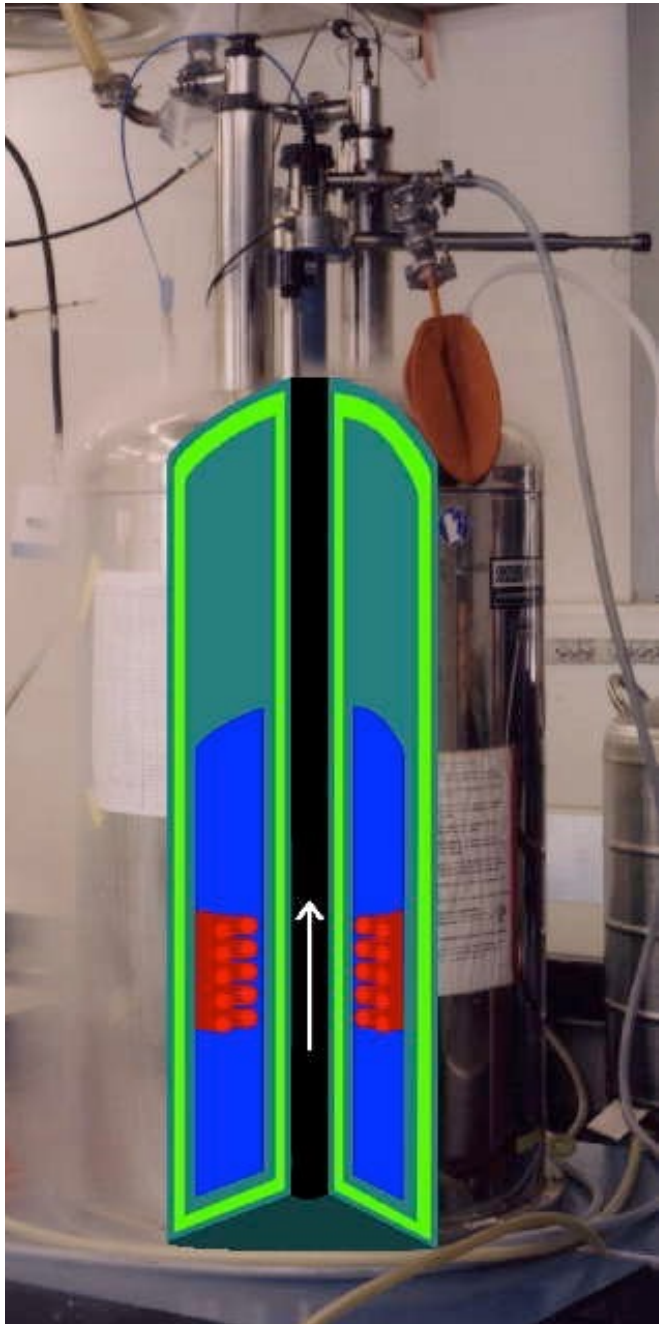
Static field: Superconductor coil
4K (He)

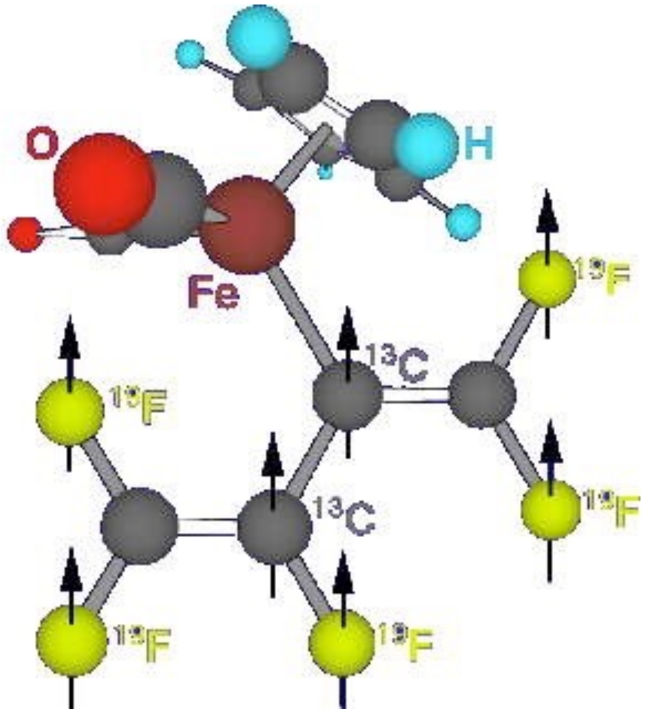
RF field: Transverse coil

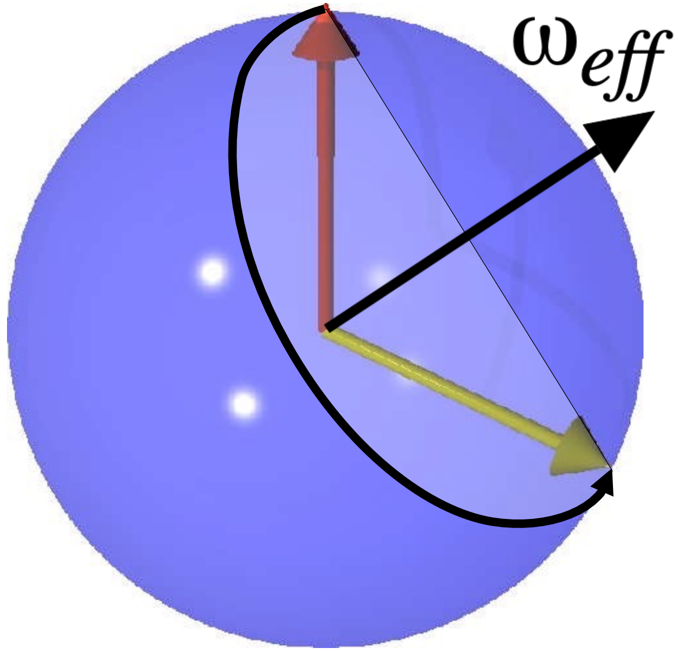
10) Liquid-State NMR



10) Liquid-State NMR







10.1 Basics of NMR

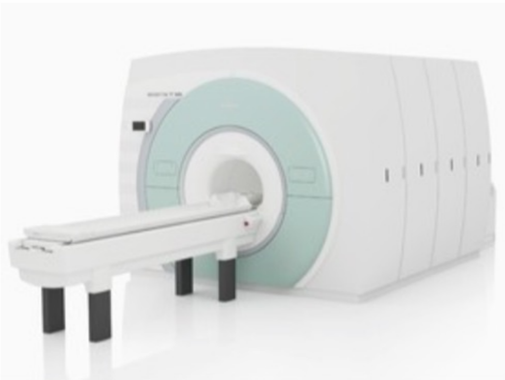
10.2 NMR as molecular quantum computer

10.3 Examples

Other Equipments NMR & MRI
MRI: Magnetic Resonance Imaging

Animal scanner 7T

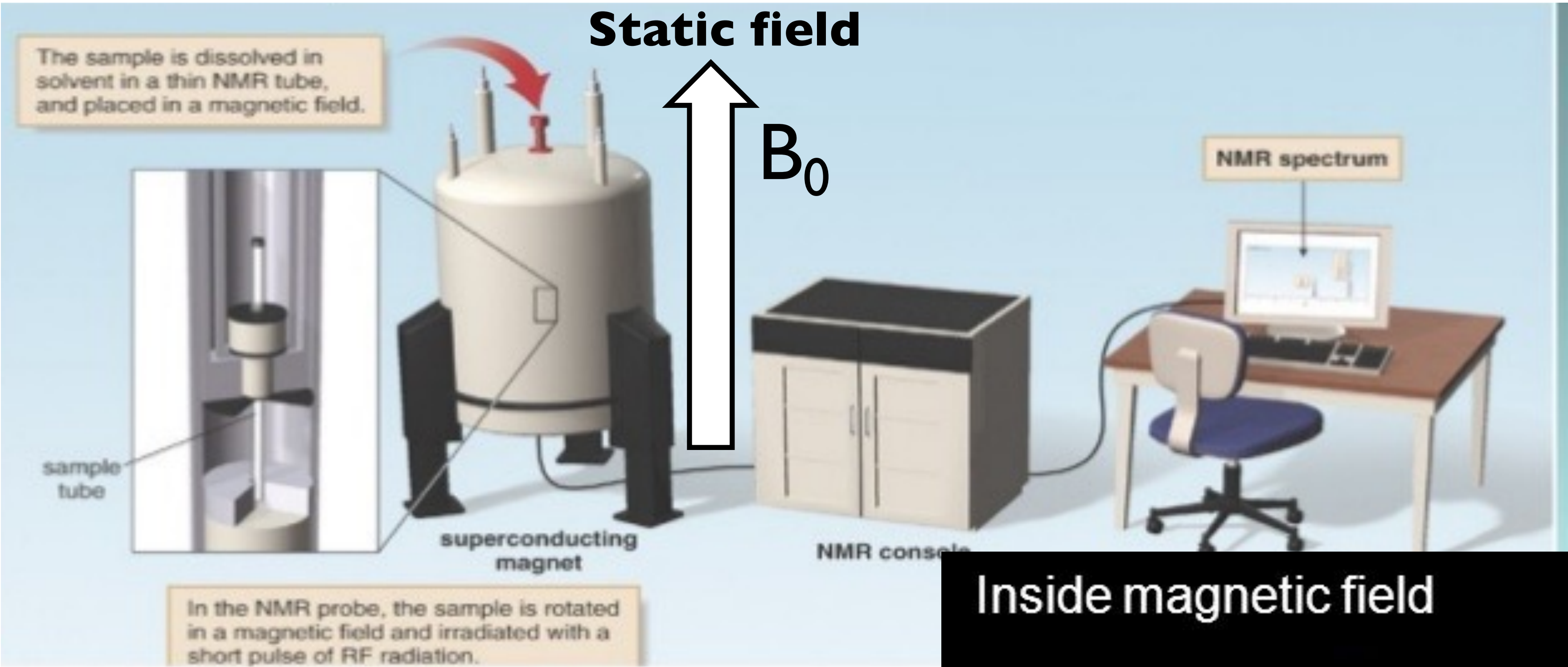
Siemens 7T

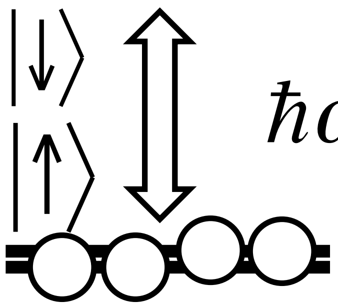


Philips 3T Ingenia

GE 3T Discovery

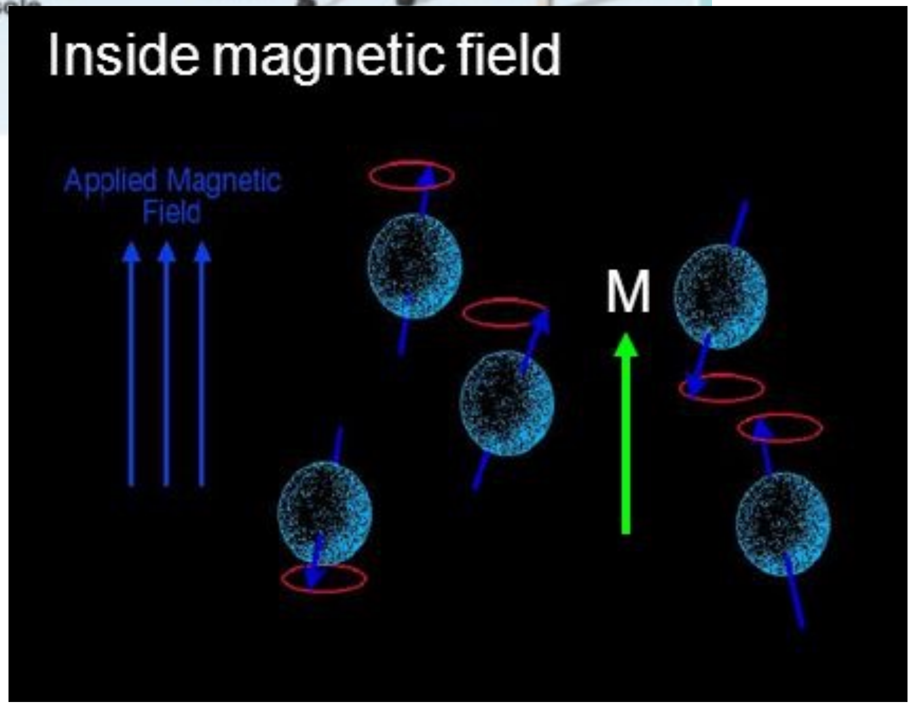
Nuclear Magnetic Resonance (NMR)





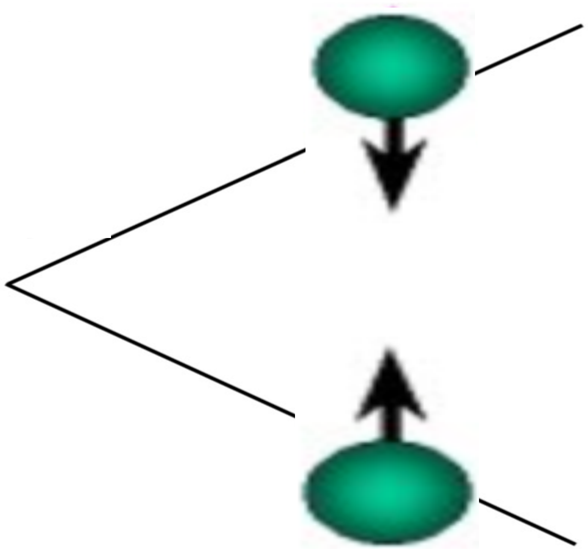
$$\hbar\omega_Z \propto B_0$$

$$\hbar\omega_Z / k_B T \sim 10^{-5}$$



10.1) Basics of NMR

Zeeman Interaction



Spin operator $I = 1/2$

Magnetic moment $\vec{\mu} = \gamma \cdot \vec{I}$

Giromagnetic factor

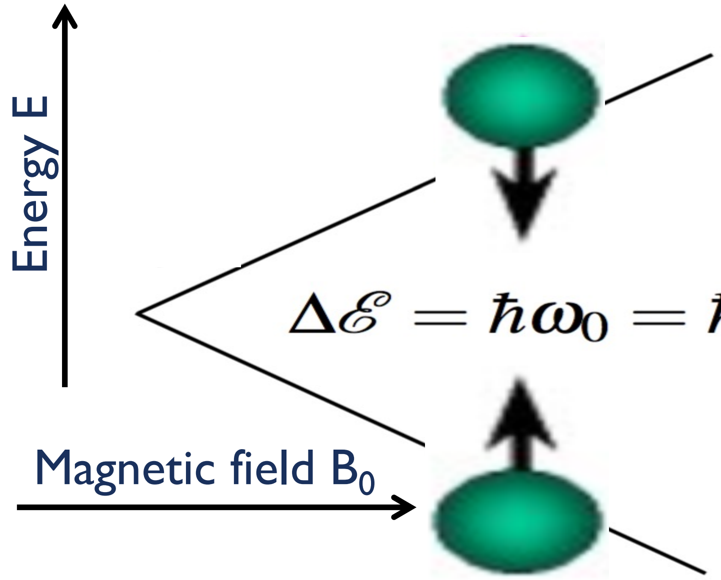
Interaction (Zeeman) Hamiltonian with the magnetic field

$$\mathscr{H} = -\vec{\mu} \cdot \vec{B} \, .$$

$$\mathscr{H} = -\gamma B_0 I_z \, .$$

10.1) Basics of NMR

Zeeman Interaction



$$\Delta\mathscr{E} = \hbar\omega_0 = \hbar\gamma B_0 \, .$$

$$\mathscr{E} = \mp\hbar\omega_0/2.$$

$$\omega_0 = \gamma B_0$$

Spins align with the field
(minimal energy)

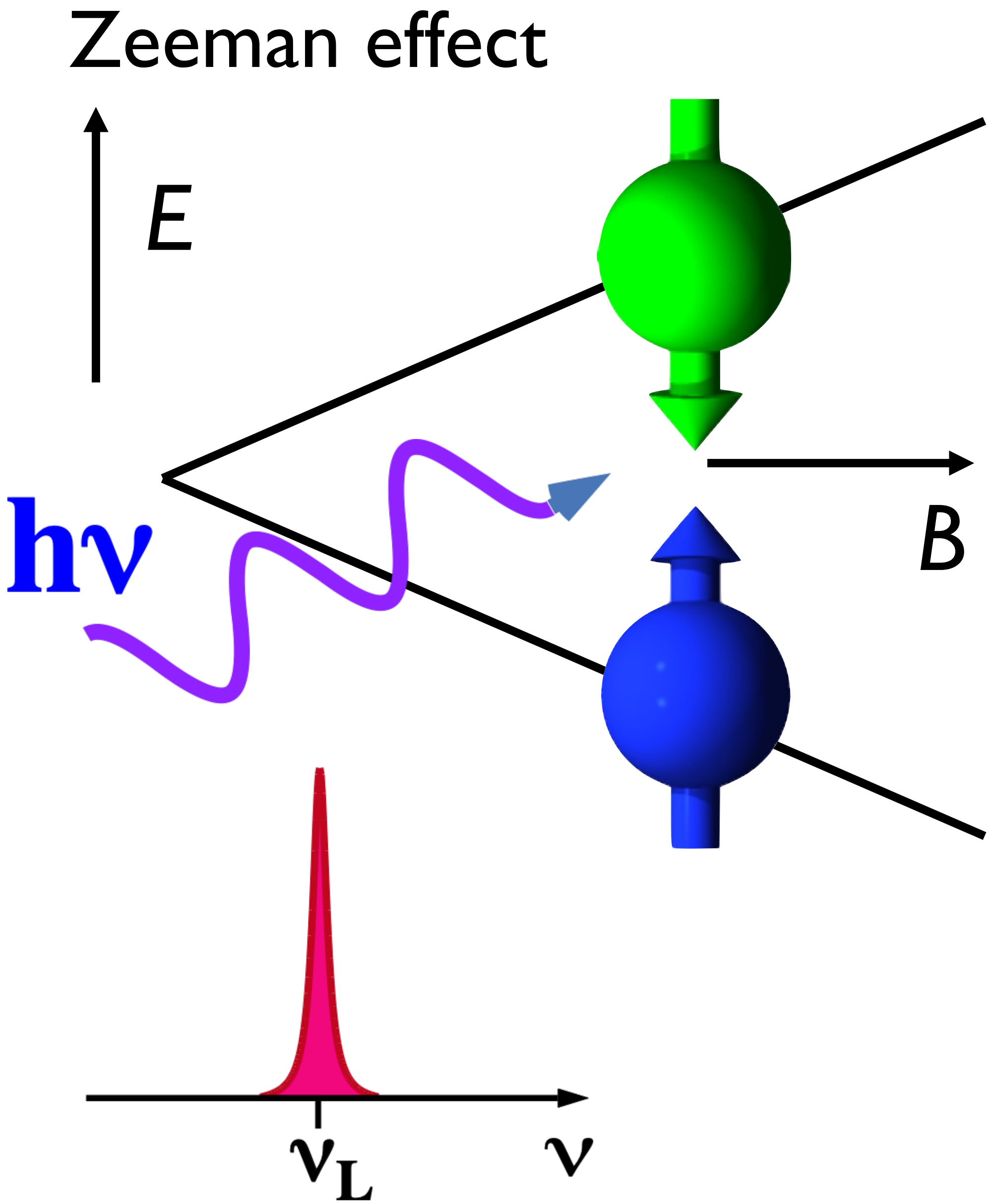
$$\mathscr{H} = -\vec{\mu} \cdot \vec{B} \, .$$

$$\mathscr{H} = -\gamma B_0 I_z \, .$$

$$I_z|I, m\rangle = \hbar m|I, m\rangle$$

$$I_z|I, \pm\frac{1}{2}\rangle = \pm\frac{\hbar}{2}|I, \pm\frac{1}{2}\rangle$$

Magnetic Resonance



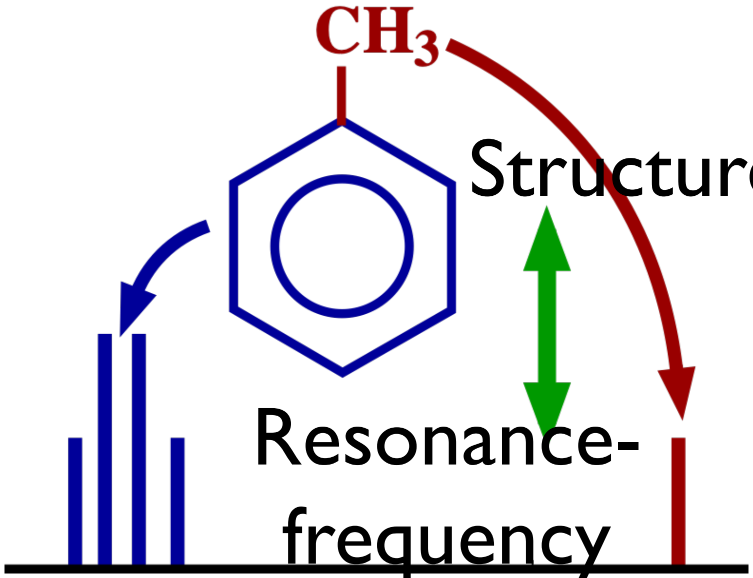
$$\mathscr{H} = -\vec{\mu} \cdot \vec{B} \,.$$

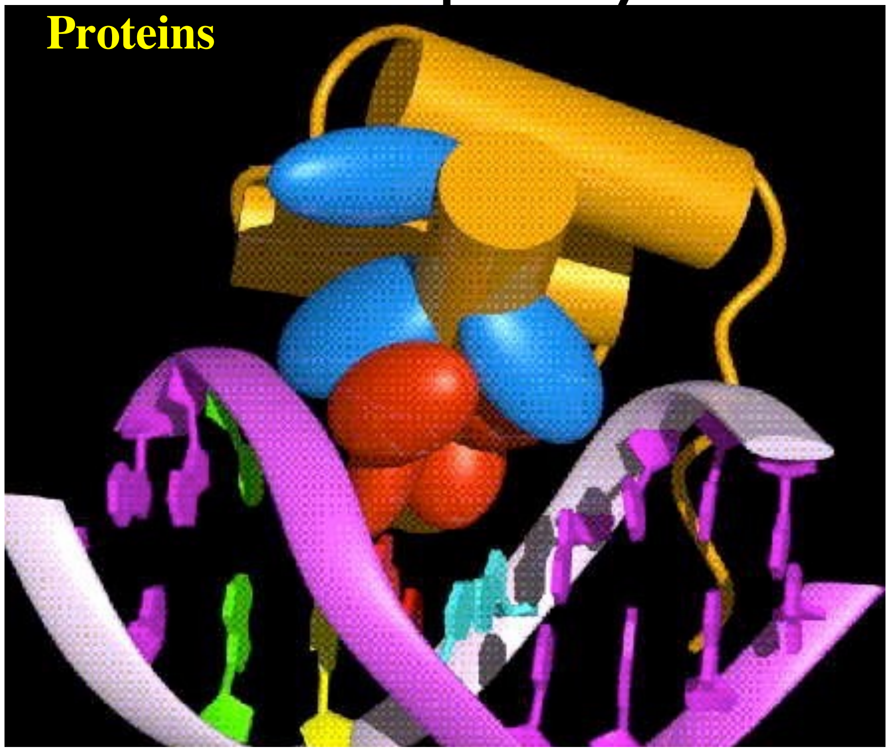
$$\mathscr{H} = -\gamma B_0 I_z \,.$$

Principle

MR measures transitions between spin states, which are excited by a radio-frequency field.

Applications of NMR

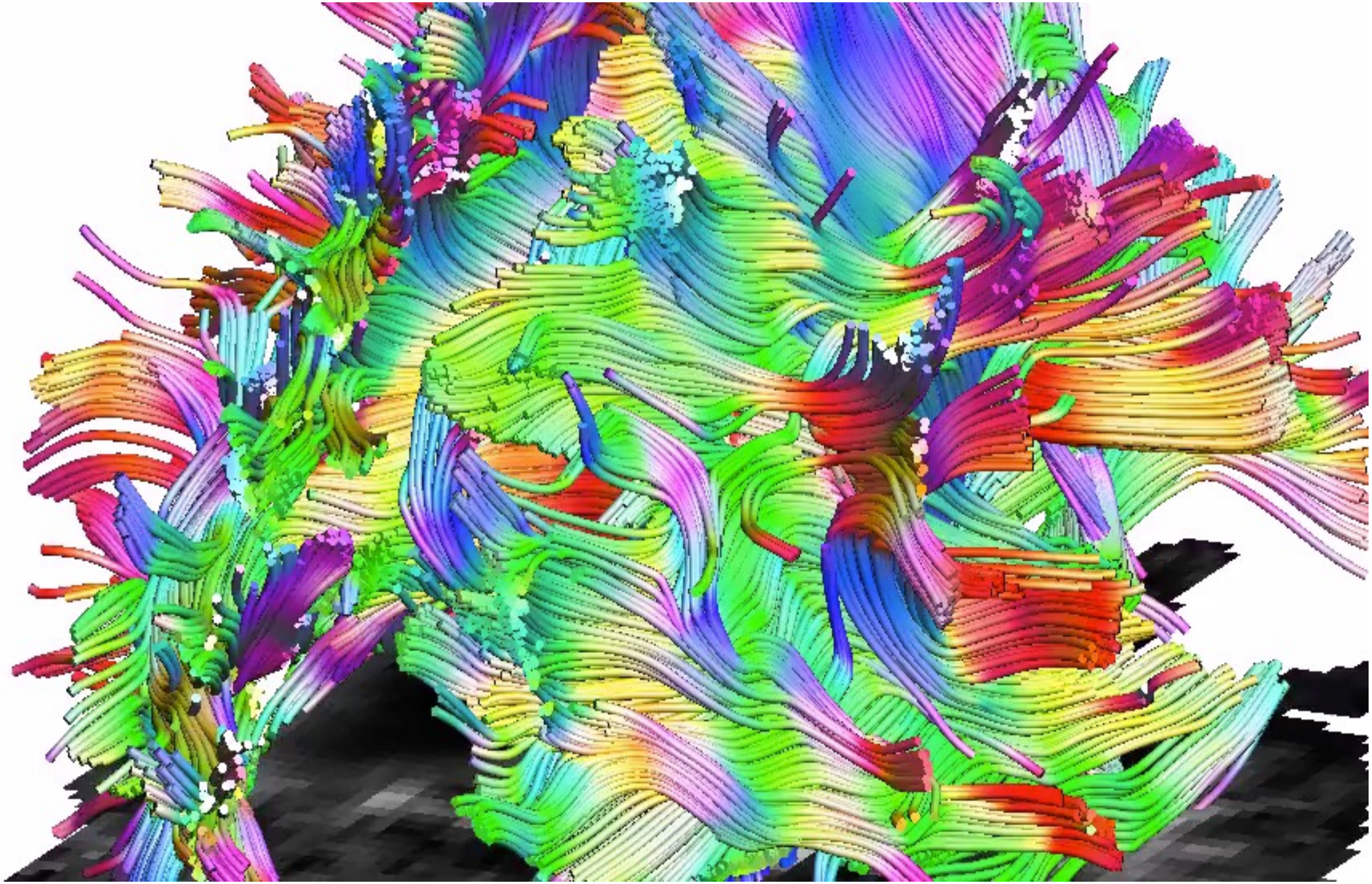




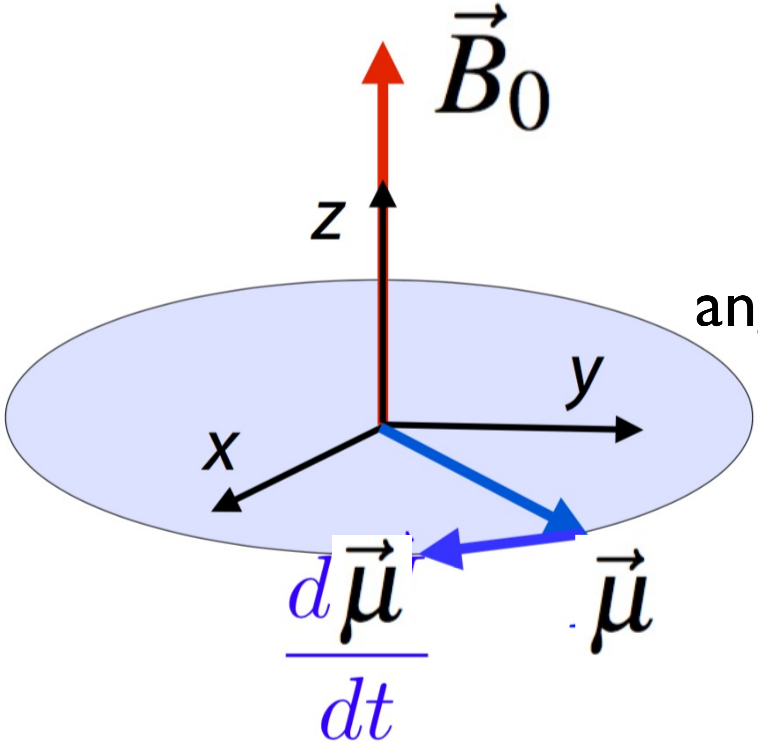
Magnetic Resonance Imaging

aka MRI, MRT

Connectivities in the Brain



Equation of motion



Torque $\vec{T} = \vec{\mu} \times \vec{B}_0.$

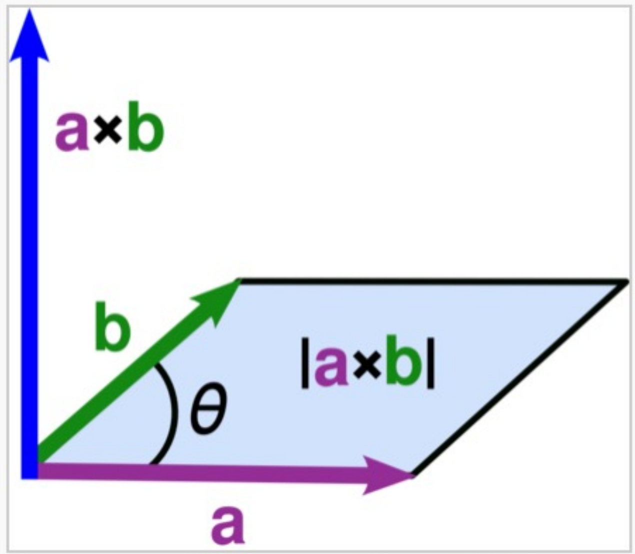
angular momentum's derivative $\frac{d\vec{I}}{dt} = \vec{T}.$

$\vec{\mu} = \gamma \cdot \vec{I}$

$$\dot{\vec{\mu}} = \gamma\vec{\mu} \times \vec{B}_0 = \vec{\mu} \times \vec{\omega}_0.$$

Ensemble average magnetic moment $\vec{M} = \frac{1}{V}\sum_i \vec{\mu}_i\,.$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \vec{\omega}_0\,.$$



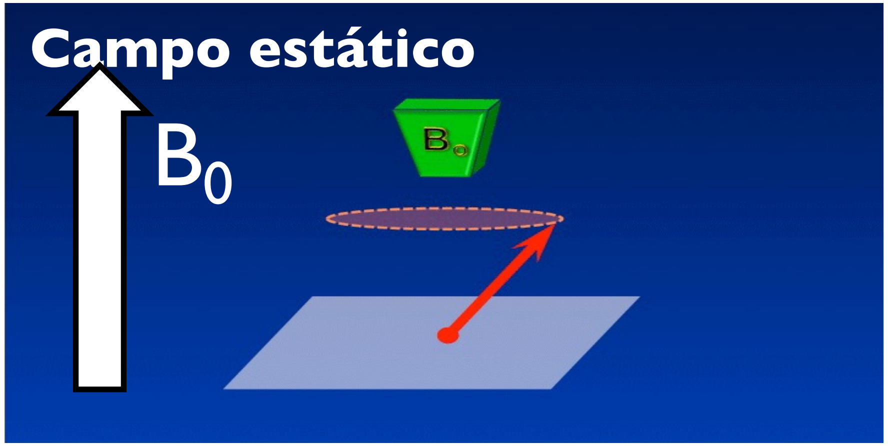
Cross product: $|\mathbf{a} \times \mathbf{b}| = a\, b\, \sin\theta.$

Equation of motion

Larmor precession: Density Matrix evolution

$$\rho = \rho_0 \frac{1}{2}\underline{\underline{1}} + \rho_x I_x + \rho_y I_y + \rho_z I_z$$

General density matrix for a ½ spin ensemble



Liouville–von Neumann equation:
Schrödinger equation version for density matrix

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\mathcal{H}, \rho] \xrightarrow{\text{Solution}} \rho(t) = U(t)\rho(0)U^{\dagger}(t)$$

Evolution operator of Zeeman interaction

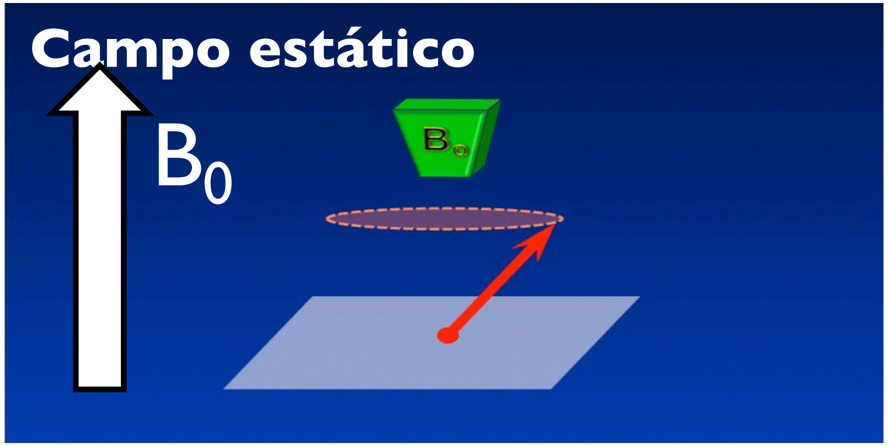
$$U(t) = \exp\left(-i\mathcal{H}t/\hbar\right) = \exp\left(i\gamma B_0 I_z t/\hbar\right)$$

Equation of motion

Larmor precession: Density Matrix evolution

$$\rho = \rho_0 \frac{1}{2}\underline{\underline{1}} + \rho_x I_x + \rho_y I_y + \rho_z I_z$$

General density matrix for a ½ spin ensemble



Liouville–von Neumann equation:

Schrödinger equation version for density matrix

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\mathcal{H}, \rho] \xrightarrow{\text{Solution}} \rho(t) = U(t)\rho(0)U^{\dagger}(t)$$

$$\rho(t) = \rho_0(t)\tfrac{1}{2}\underline{\underline{1}} + \rho_x(t) I_x + \rho_y(t) I_y + \rho_z(t) I_z$$

$$\boxed{\begin{aligned}
\rho_0(t) &= 1 \\
\rho_x(t) &= \rho_x(0)\cos(\omega_0 t) + \rho_y(0)\sin(\omega_0 t) \\
\rho_y(t) &= \rho_y(0)\cos(\omega_0 t) - \rho_x(0)\sin(\omega_0 t) \\
\rho_z(t) &= \rho_z(0)
\end{aligned}}$$

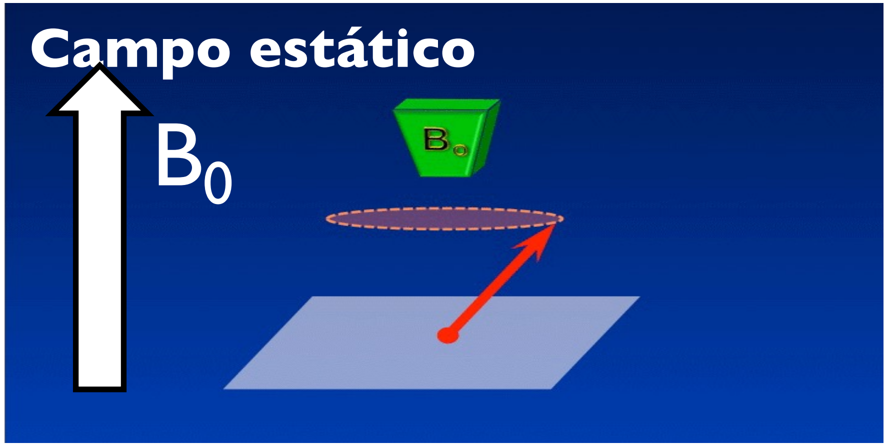
$$\boxed{M_{x,y,z} = N\gamma \mathrm{Tr}\left(I_{x,y,z}\rho(t)\right)}$$

Measurements: Coherence

Larmor precession: Density Matrix evolution

$$\rho = \rho_0 \frac{1}{2}\underline{\underline{1}} + \rho_x I_x + \rho_y I_y + \rho_z I_z$$

General density matrix for a ½ spin ensemble



Liouville–von Neumann equation:

Schrödinger equation version for density matrix

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\mathcal{H}, \rho] \xrightarrow{\text{Solution}} \rho(t) = U(t)\rho(0)U^{\dagger}(t)$$

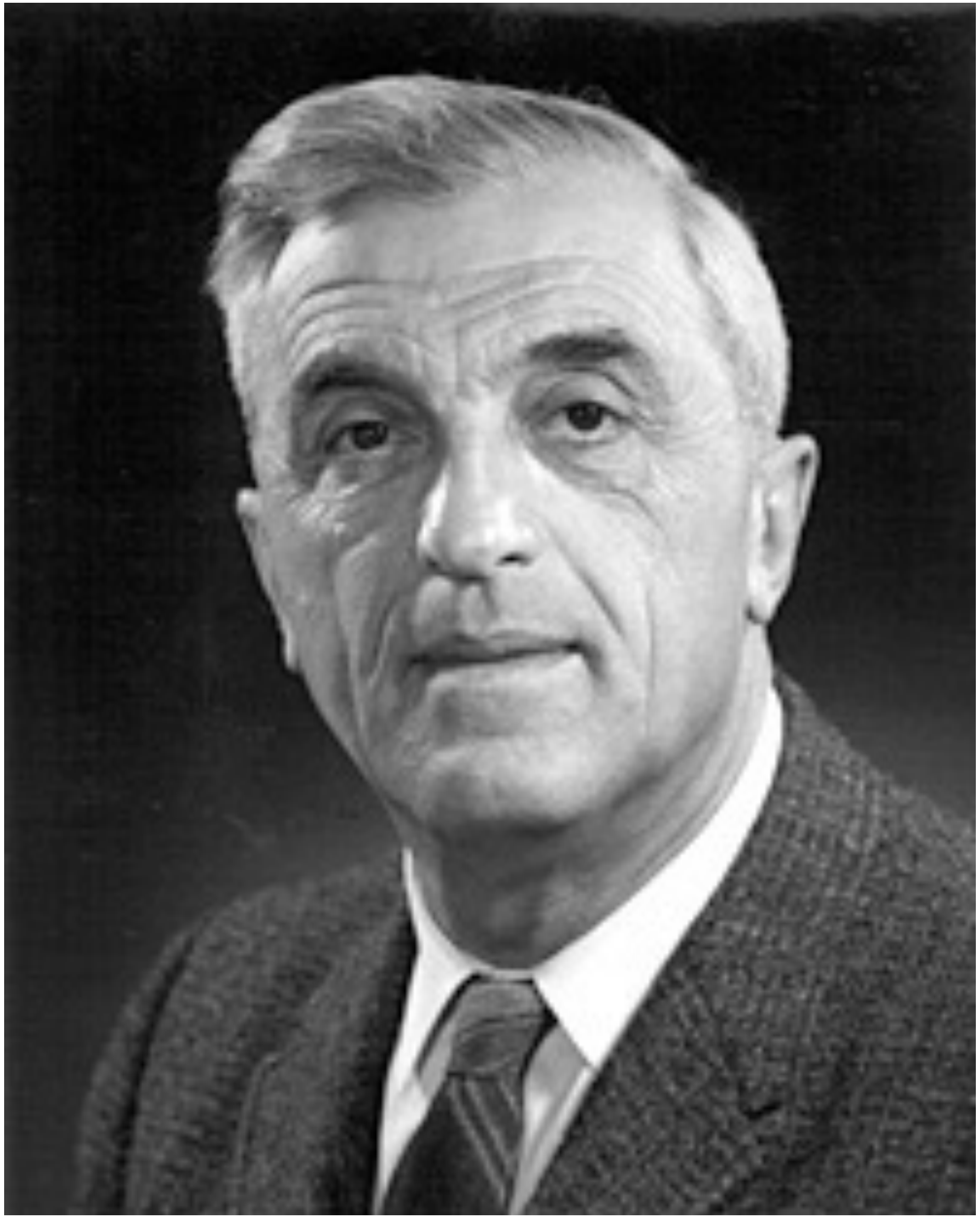
$$\rho(t) = \rho_0(t)\tfrac{1}{2}\underline{\underline{1}} + \rho_x(t)I_x + \rho_y(t)I_y + \rho_z(t)I_z$$

$$\boxed{\begin{aligned} M_x &= M_{xy}\cos(\omega_0 t + \phi) \\ M_y &= -M_{xy}\sin(\omega_0 t + \phi) \\ M_z &= const. \end{aligned}}$$

Gonzalo A. Alvarez

$$\boxed{\frac{d\vec{M}}{dt} = \vec{M} \times \vec{\omega}_0 \,.}$$

Bloch Equations



Equation of motion

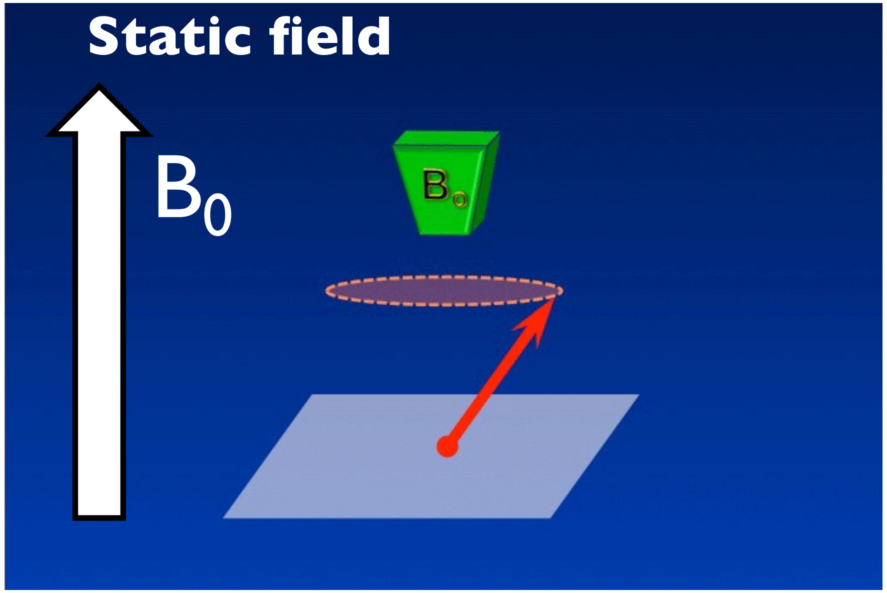
$$\frac{d}{dt}\langle \mathbf{S}_x \rangle = -\omega_L \langle \mathbf{S}_y \rangle$$
$$\frac{d}{dt}\langle \mathbf{S}_y \rangle = \omega_L \langle \mathbf{S}_x \rangle$$
$$\frac{d}{dt}\langle \mathbf{S}_z \rangle = 0.$$

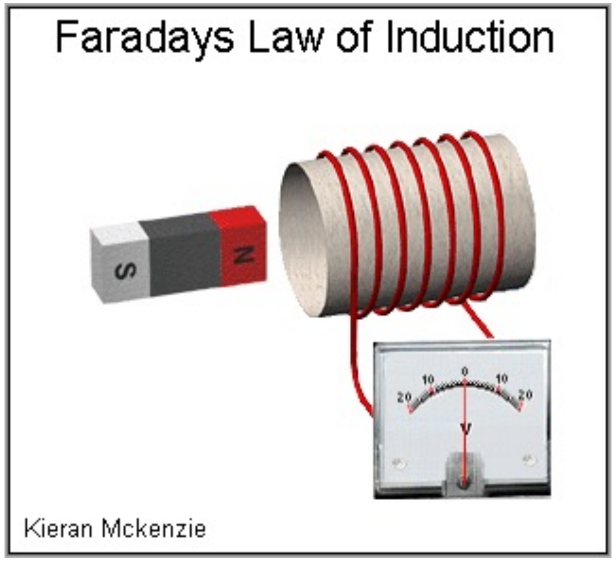
Solution

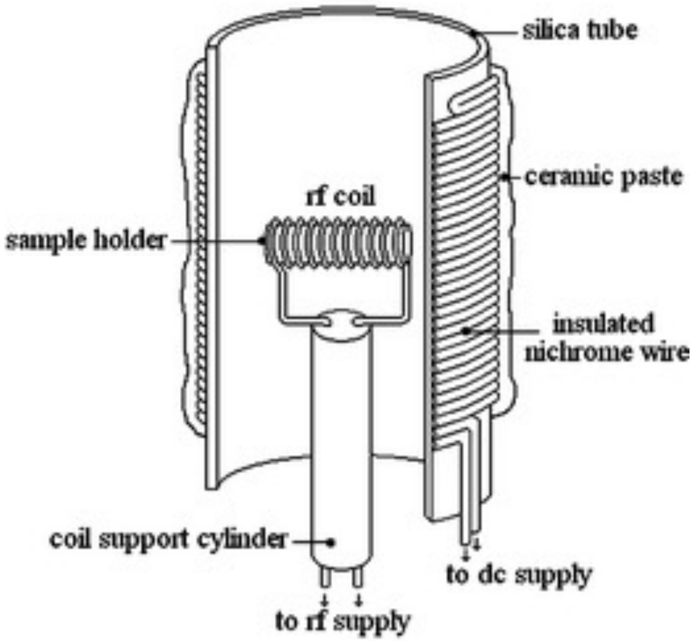
$$\langle \mathbf{S}_x \rangle(t) = S_{xy}(0)\cos(\omega_L t - \phi)$$
$$\langle \mathbf{S}_y \rangle(t) = S_{xy}(0)\sin(\omega_L t - \phi)$$
$$\langle \mathbf{S}_z \rangle(t) = S_z(0)$$

Felix Bloch
(1905 - 1983)

Signal Detection

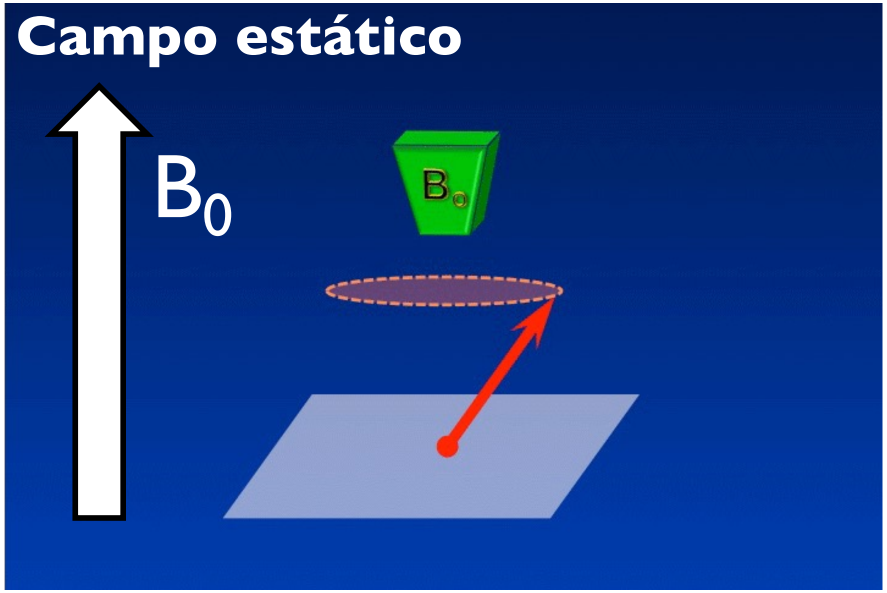


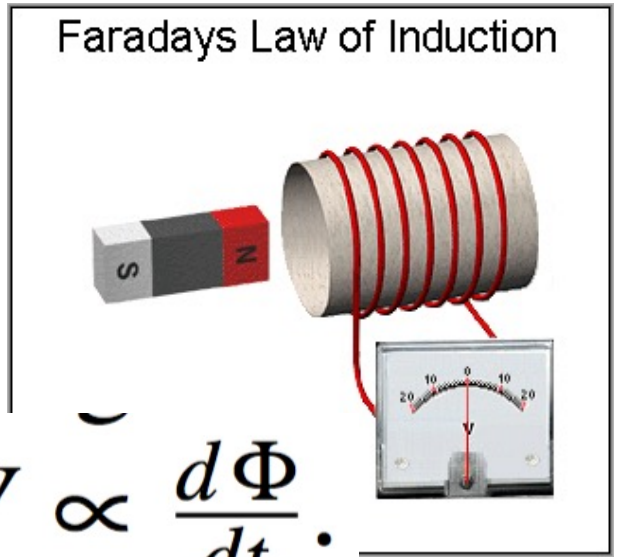




Precessing spins = rotating magnetisation

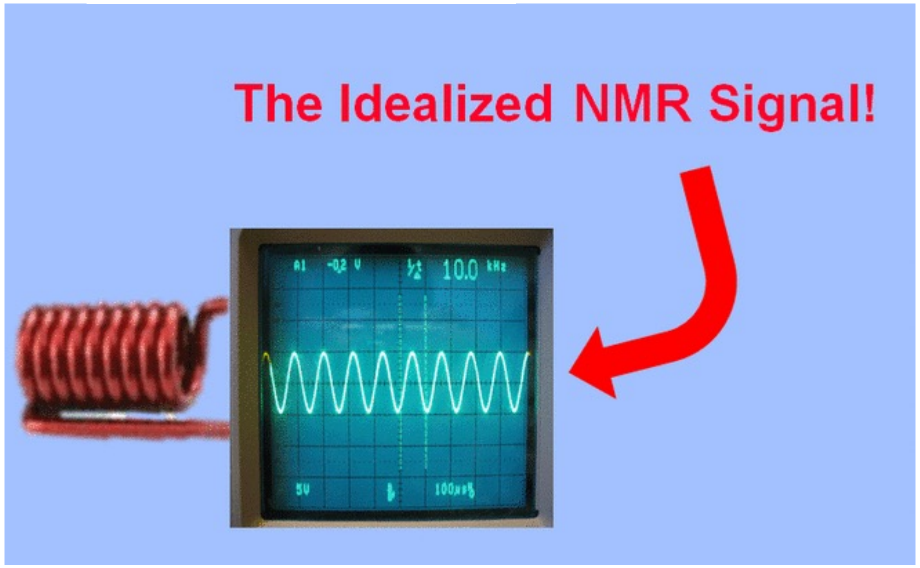
Signal Detection





$$V \propto \frac{d\Phi}{dt}.$$

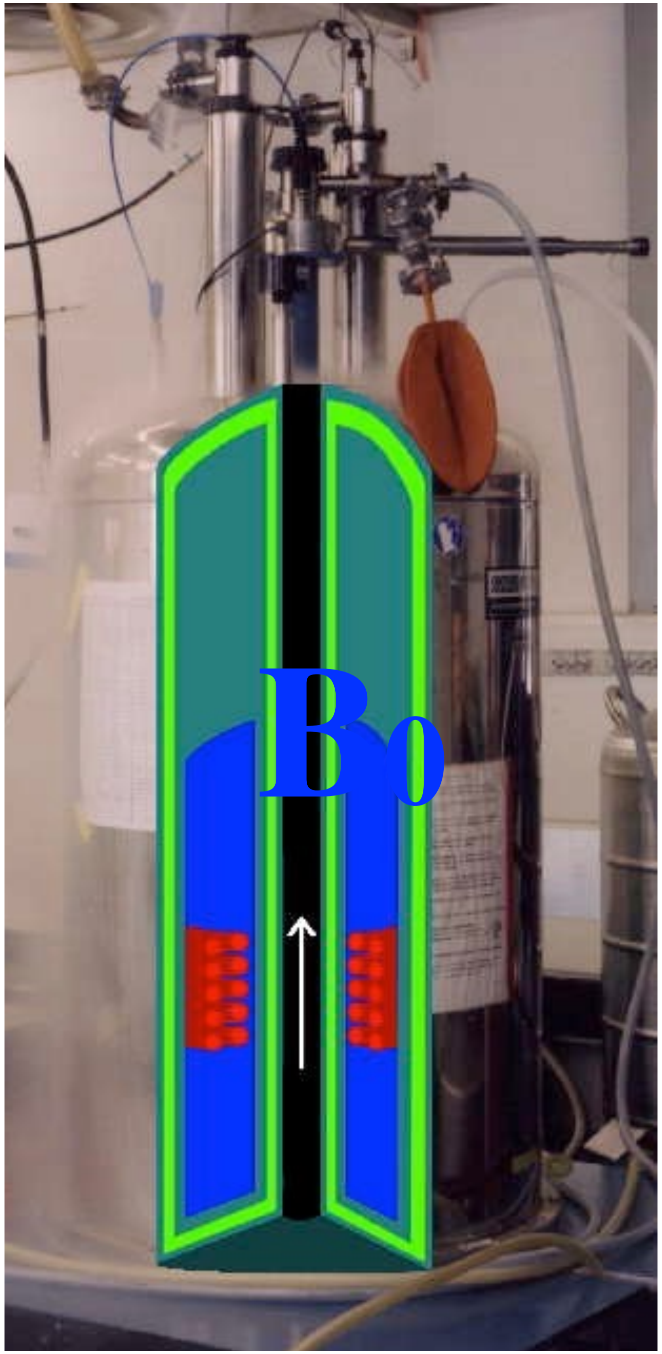
Change of
flux induces
voltage

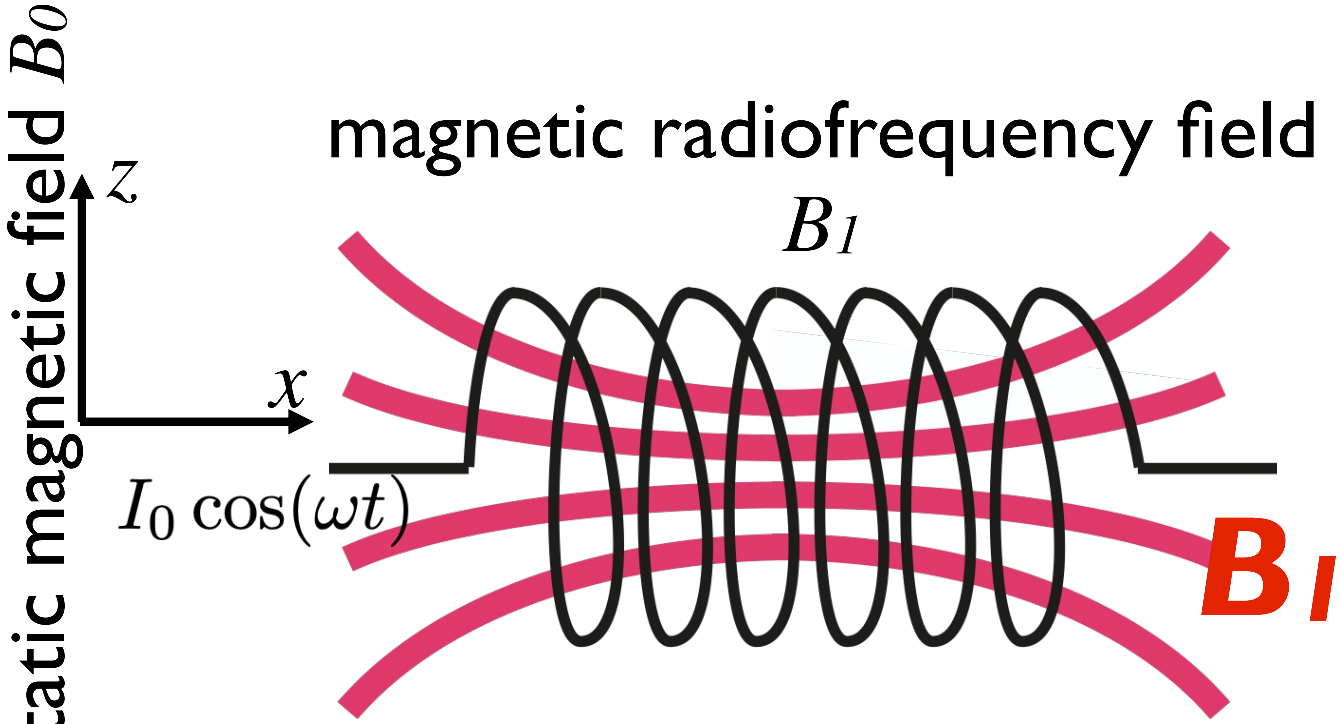


$$\vec{S}(t) = (S_x \cos \Omega_L t + S_y \sin \Omega_L t)$$

$$\vec{s}(t) \propto \cos \Omega_L t$$

Excitation: spin state control



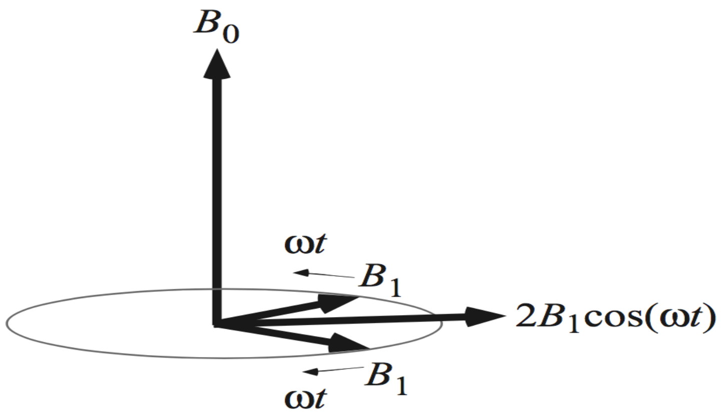


$$\vec{B}_{rf}(t) \; = \; 2B_1 \cos(\omega t) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \; B_1 \begin{pmatrix} \cos(\omega t) \\ \sin(\omega t) \\ 0 \end{pmatrix} + B_1 \begin{pmatrix} \cos(\omega t) \\ -\sin(\omega t) \\ 0 \end{pmatrix}$$

Spin control
→ RF magnetic fields

Spin evolution with a time dependent field (RF field)



$$\mathcal{H}_{\text{lab}} = -\gamma B_0 I_z - 2\gamma B_1 \cos(\omega t)\, I_x$$

$$\mathcal{H}_{\text{lab}} = -\gamma B_0 I_z - \gamma B_1 \cos(\omega t) I_x + \gamma B_1 \cos(\omega t) I_z - \gamma B_1 \cos(\omega t) I_x - \gamma B_1 \cos(\omega t) I_z$$

$$\boxed{\mathcal{H}_{\text{lab}} = -\gamma B_0 I_z - \gamma B_1 \exp(i\omega t I_z)\, I_x \exp(-i\omega t I_z) - \gamma B_1 \exp(-i\omega t I_z)\, I_x \exp(i\omega t I_z)}$$

Hint !

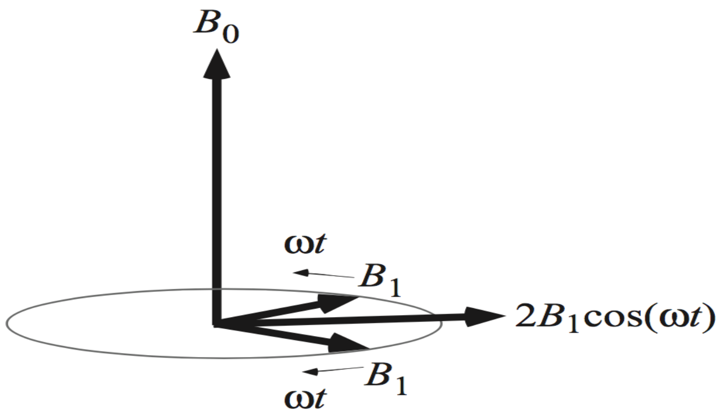
$$U(\hat{\mathbf{n}}, \theta) = e^{-i\theta \hat{\mathbf{n}}\cdot\boldsymbol{\sigma}/2} = \cos\frac{\theta}{2} - i(\hat{\mathbf{n}}\cdot\boldsymbol{\sigma})\sin\frac{\theta}{2},$$

$$\exp\{\mathrm{i}I_z\phi/\hbar\}\, I_x \exp\{-\mathrm{i}I_z\phi/\hbar\} = I_x\cos\phi - I_y\sin\phi$$

$$\exp\{-\mathrm{i}I_z\phi/\hbar\}\, I_x \exp\{\mathrm{i}I_z\phi/\hbar\} = I_x\cos\phi + I_y\sin\phi$$

Spin control
→ RF magnetic fields

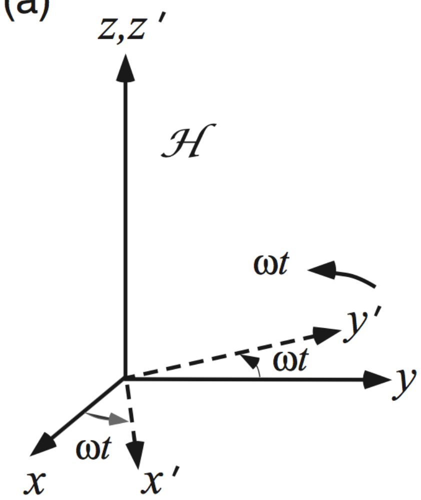
Spin evolution with a time dependent field (RF field)



$$\mathcal{H}_{\mathrm{lab}} = -\gamma B_0 I_z - 2\gamma B_1 \cos(\omega t)\, I_x$$

$$\boxed{\mathcal{H}_{\mathrm{lab}} = -\gamma B_0 I_z - \gamma B_1 \exp(i\omega t I_z)\, I_x \exp(-i\omega t I_z) - \gamma B_1 \exp(-i\omega t I_z)\, I_x \exp(i\omega t I_z)}$$

(a)



Lab frame

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}\left[\mathcal{H}_{\mathrm{lab}}, \rho\right]$$

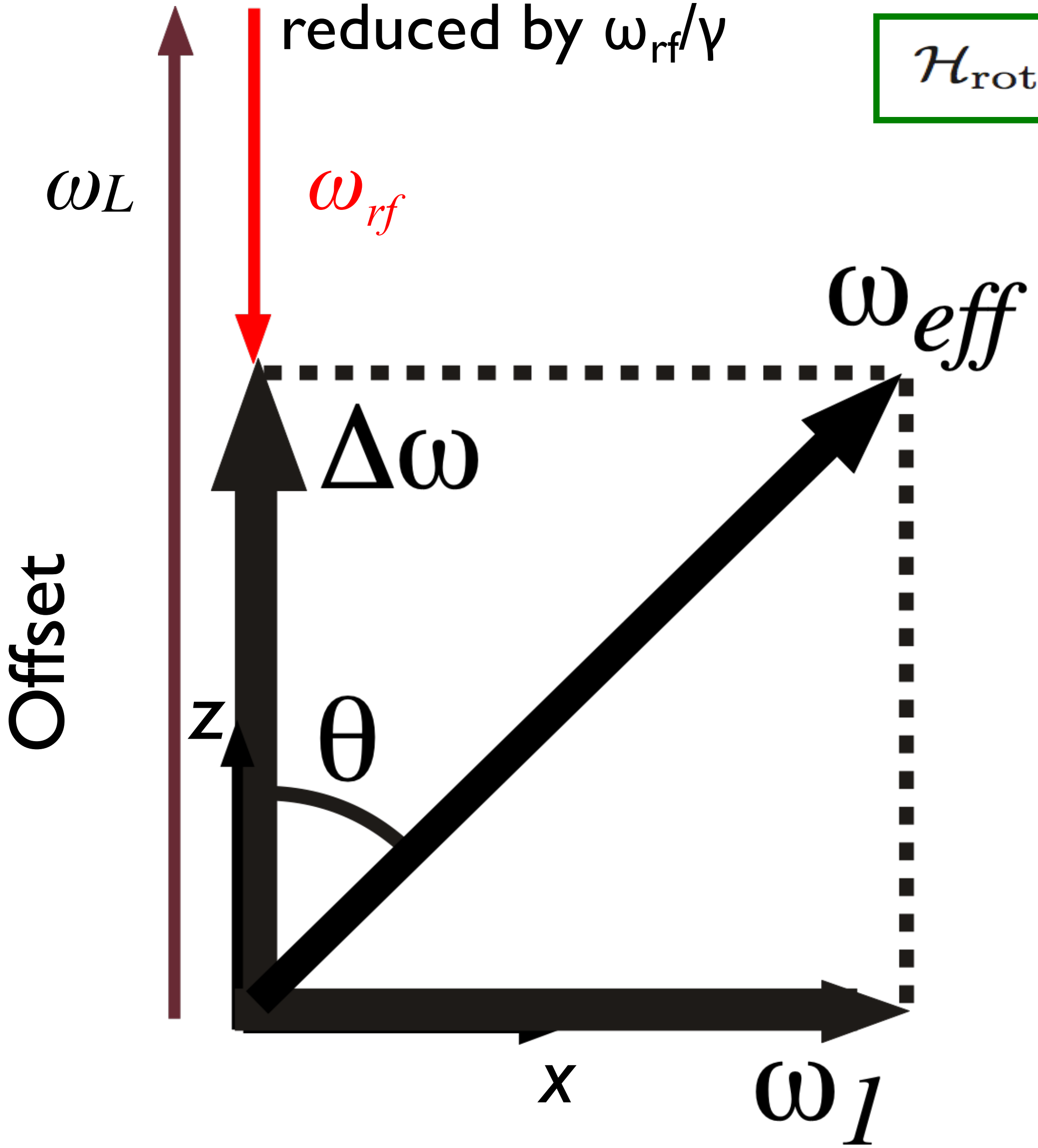
Rotating frame

$$\frac{\partial \rho^{\mathrm{rot}}}{\partial t} = -\frac{i}{\hbar}\left[\mathcal{H}_{\mathrm{rot}}, \rho^{\mathrm{rot}}\right]$$

$\mathcal{H}_{\mathrm{rot}} = -$ $\boxed{\mathcal{H}_{\mathrm{rot}} = -\gamma\left(B_0 - \omega/\gamma\right) I_z - \gamma B_1 I_x}$ xp $(i2\omega t I_z)$

Effective Field

z-component of magnetic field is reduced by ω_{rf}/γ



$$\mathcal{H}_{\text{rot}} = -\gamma\left(B_0 - \omega/\gamma\right) I_z - \gamma B_1 I_x$$

$$\Delta\omega = \omega_0 - \omega_{rf} \qquad \omega_1 = \gamma B_1$$

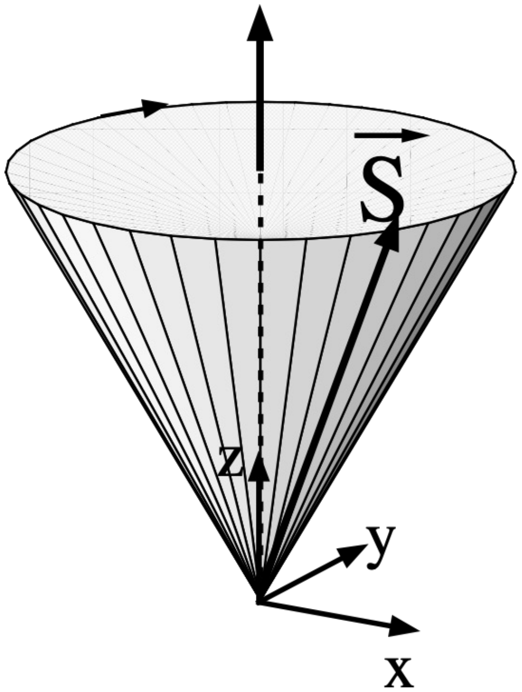
$$\vec{\omega}_{eff} = \gamma \vec{B}_{eff} = \begin{pmatrix} \omega_1 \\ 0 \\ \Delta\omega \end{pmatrix},$$

Arbitrary directions are possible by adjusting frequency and phase of RF field

x-component given by RF field amplitude

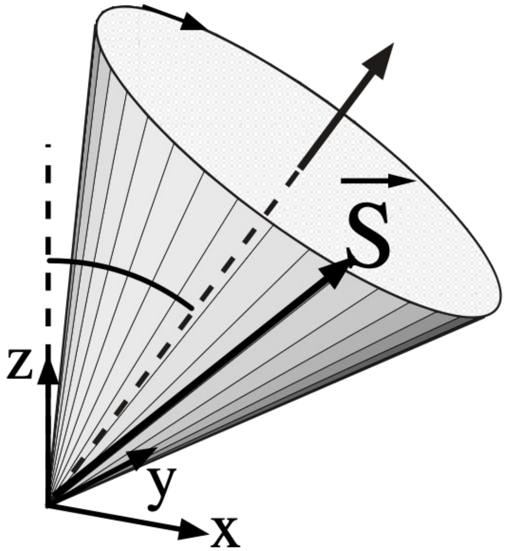
Precession : Special Cases

$$\Delta\omega \neq 0$$
$$\omega_1 = 0$$

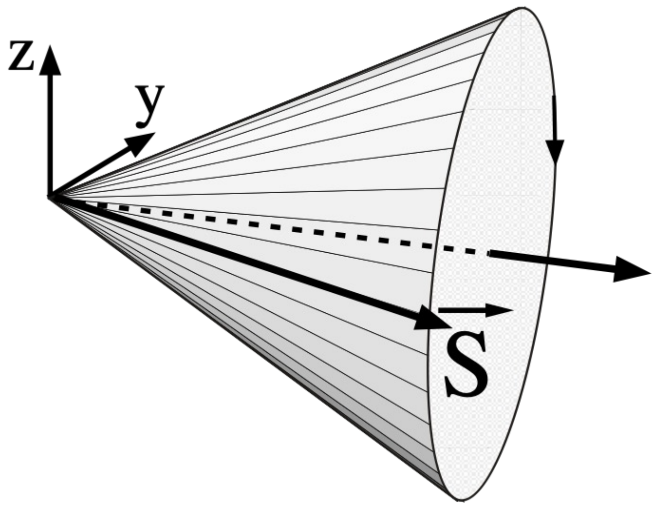


Free precession

$$\Delta\omega \neq 0$$
$$\omega_1 \neq 0$$



$$\Delta\omega = 0$$
$$\omega_1 \neq 0$$



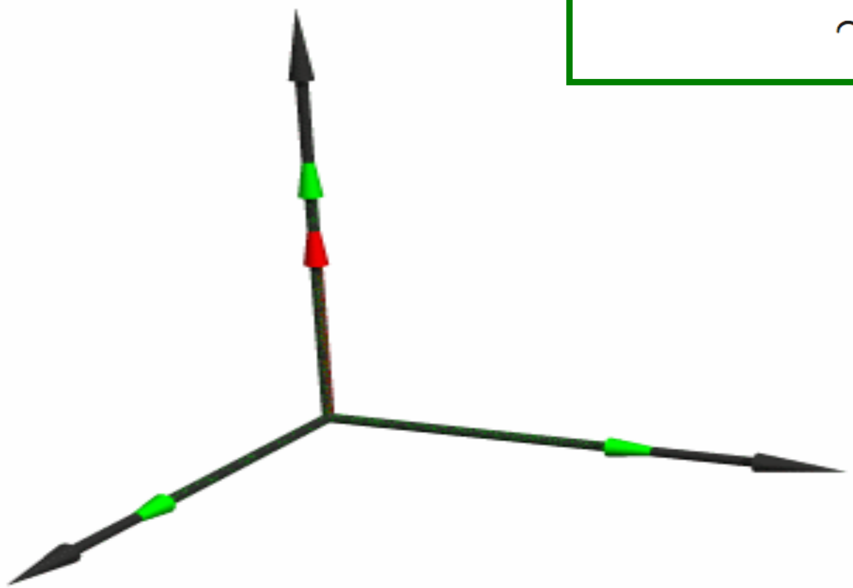
Resonant excitation

Spin control
→ RF magnetic fields

Spin evolution with a time dependent field (RF field)

$$\omega_1 = \gamma B_1$$

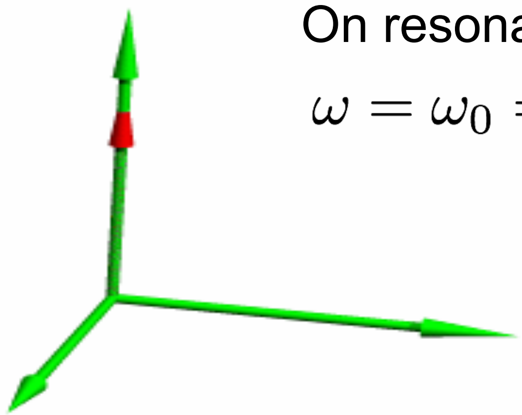
$$\boxed{\begin{aligned}\rho^{\mathrm{rot}}(t) &\sim \exp\left(i\omega_1 t I_x\right) I_z \exp\left(-i\omega_1 t I_x\right) \\ &\sim I_z \cos\left(\omega_1 t\right) + I_y \sin\left(\omega_1 t\right)\end{aligned}}$$



Lab frame

On resonance RF

$$\omega = \omega_0 = \gamma B_0$$

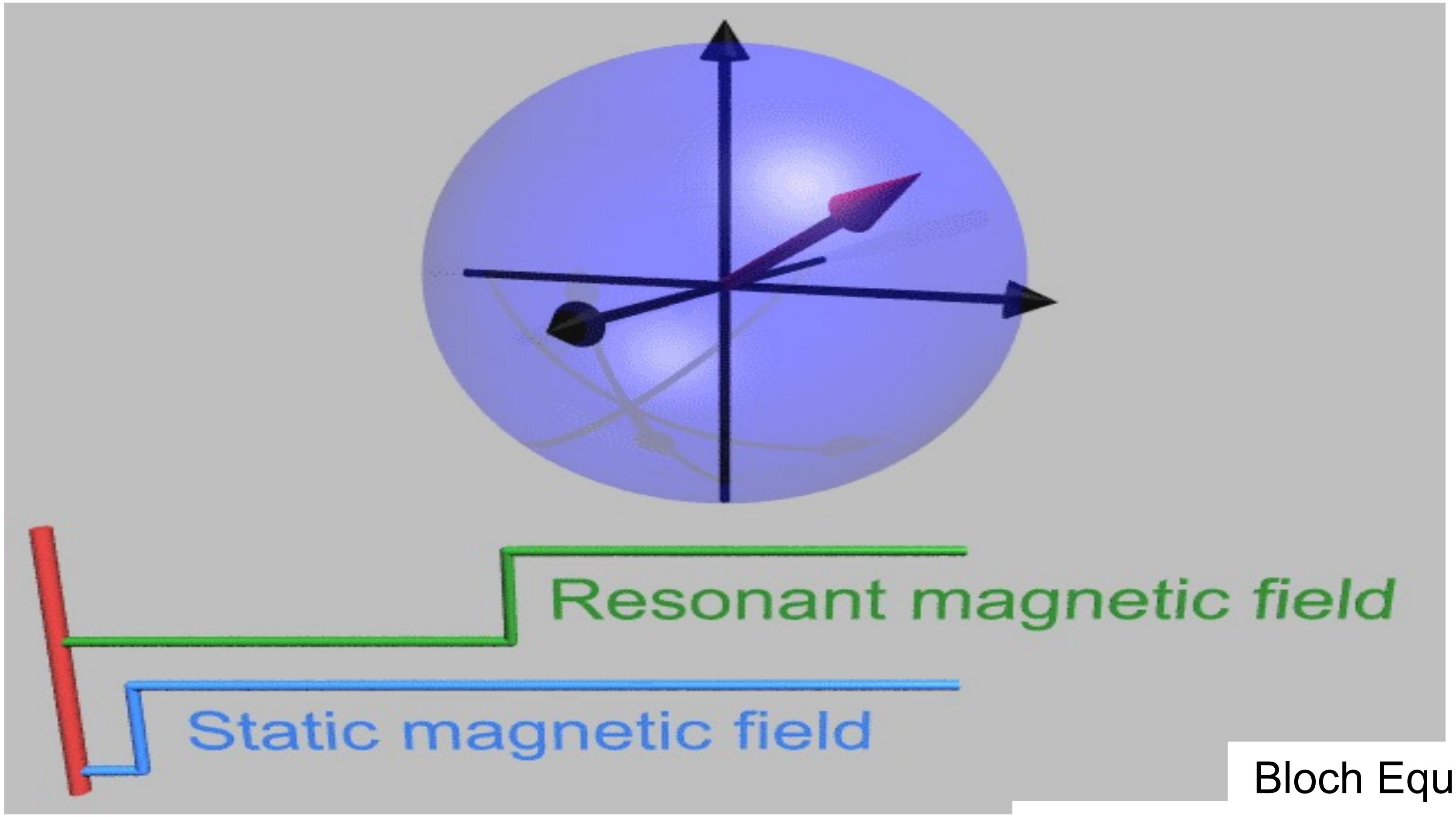


Rotating frame

$$\rho^{\mathrm{lab}}(t) \sim I_z \cos\left(\omega_1 t\right) + I_y \cos\left(\omega_0 t\right) \sin\left(\omega_1 t\right) + I_x \sin\left(\omega_0 t\right) \sin\left(\omega_1 t\right)$$

Spin control
→ RF magnetic fields

Spin evolution in different frames



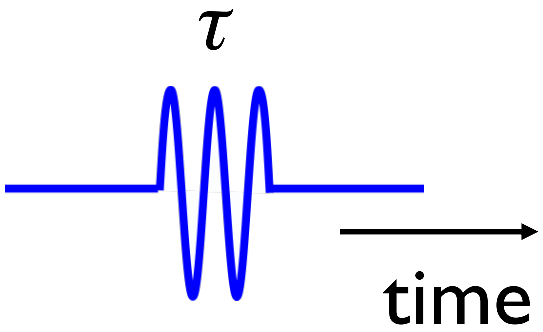
Bloch Equation

Nucleos: T_1~seg, T^*_2~100us-1ms (solids), T_2~500ms liquids

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}$$ +Relaxation

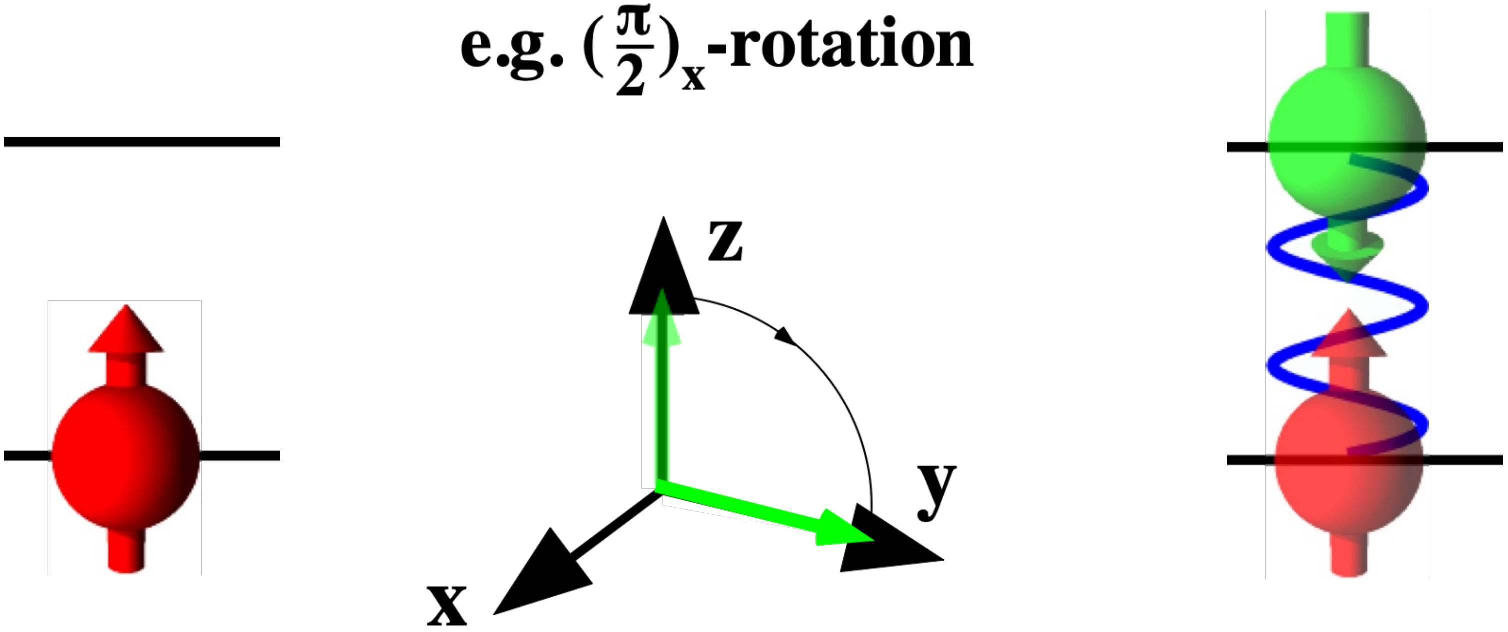
RF Pulses

$\pi/2$ pulse

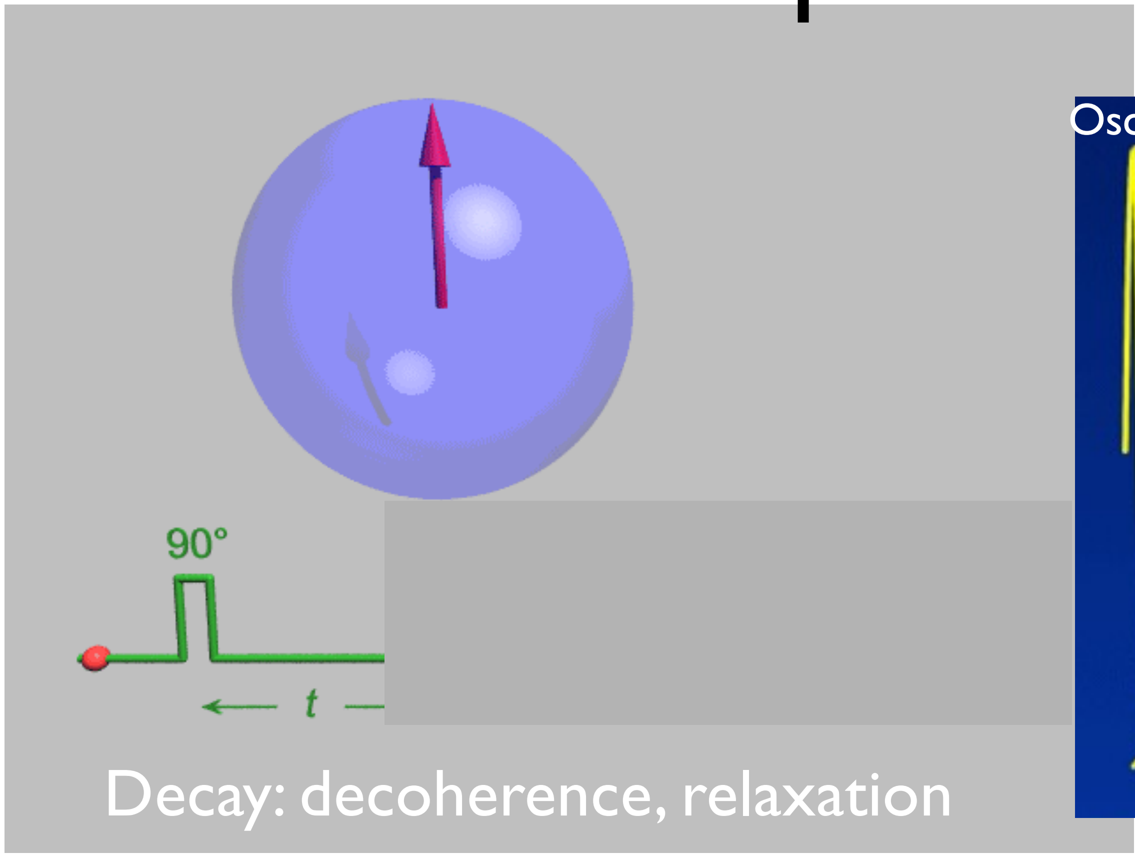


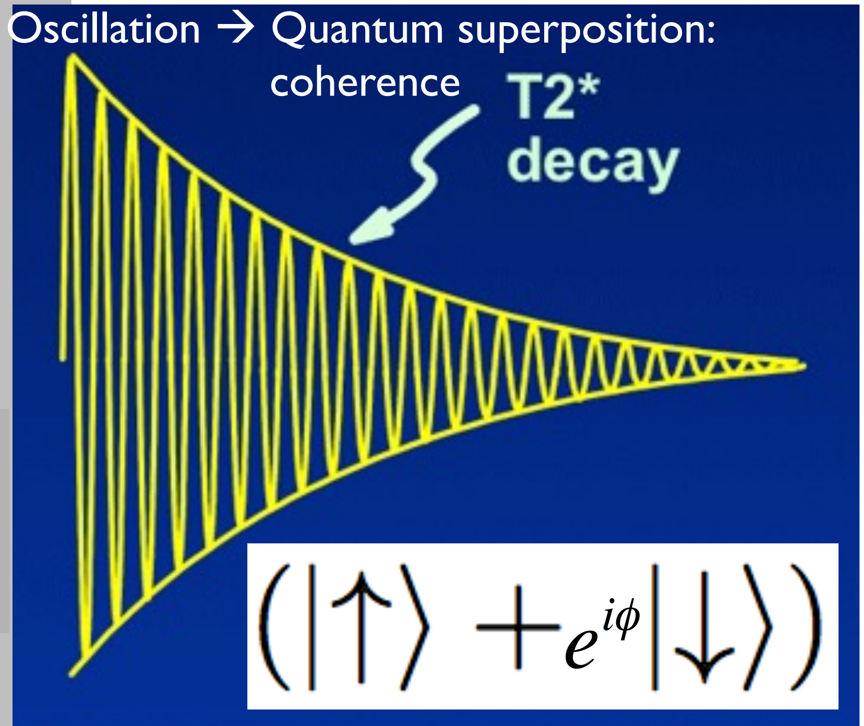
Choose $\omega_{eff}\tau = \pi/2$

e.g. $(\frac{\pi}{2})_x$-rotation



Pulse sequences: 1 pulse





Inhomogeneous Magnetic Field → (T^*_2)
Effective due to spin interactions (T_2)

Quantum Computing

Dieter Suter
Gonzalo A. Álvarez, Analía Zwick

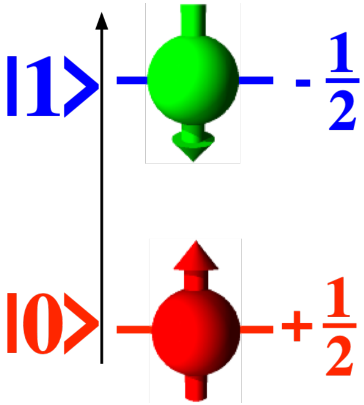
Contents

Contact:
Gonzalo A. Alvarez: gonzalo.alvarez@conicet.gov.ar
gonzalo.a.alvarez@gmail.com
Analia Zwick: analia.zwick@gmail.com
Web-page: https://www.famaf.unc.edu.ar/~galvarez/

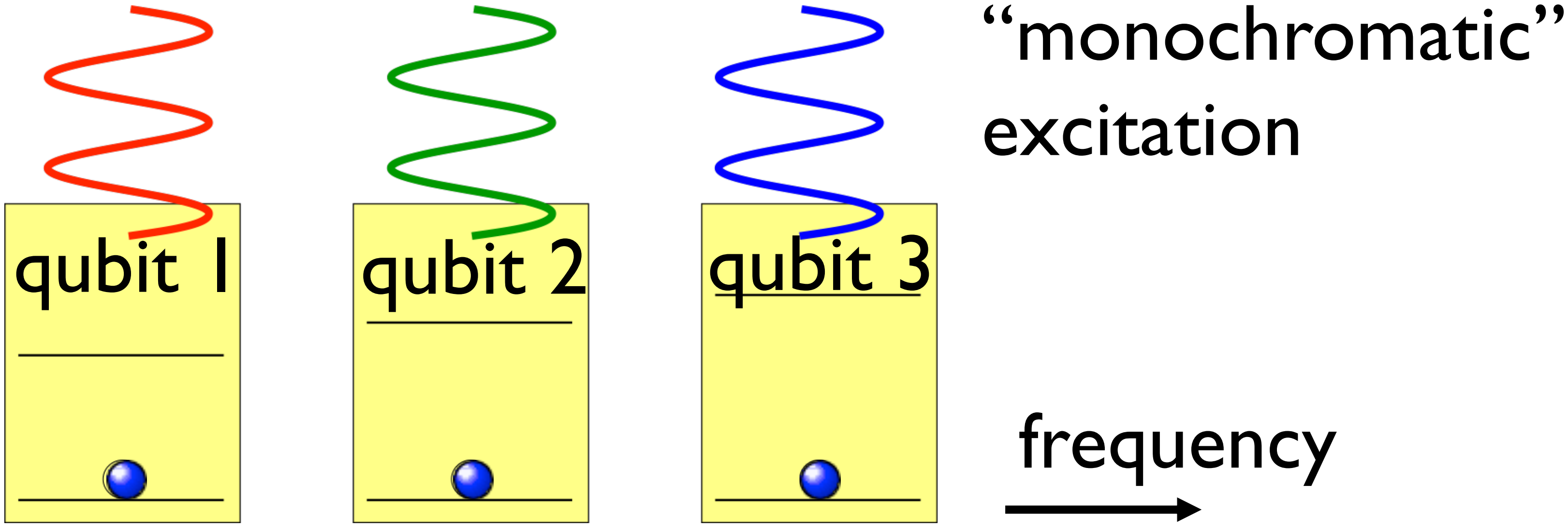
10.2 NMR as a Molecular Quantum Computer

Spin 1/2 as Qubit

Spin 1/2 Qubit

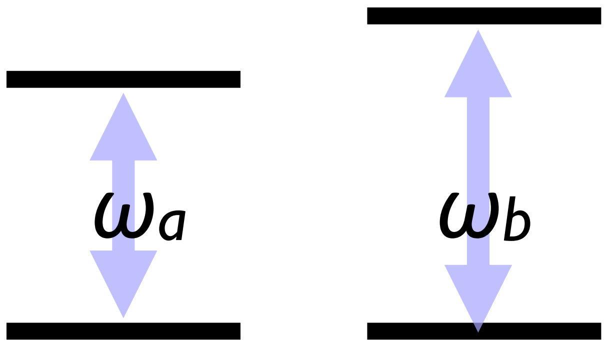


Liquid-state NMR quantum computer



Selective Excitation

Qubit a   Qubit b

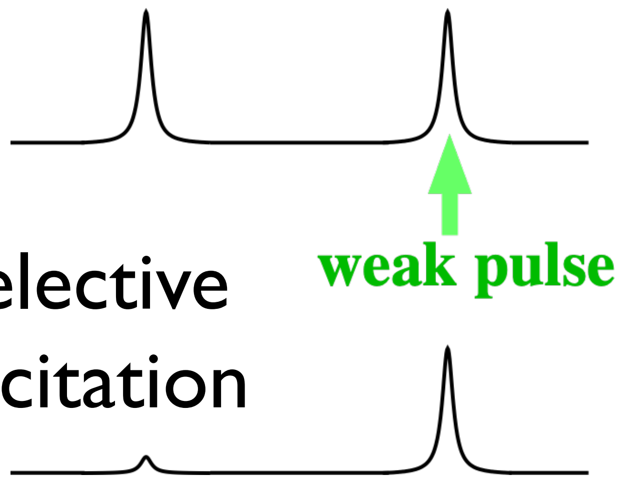


Condition for selective excitation:

$$|\omega_a - \omega_b|\tau \gg 1$$

Nonselective excitation

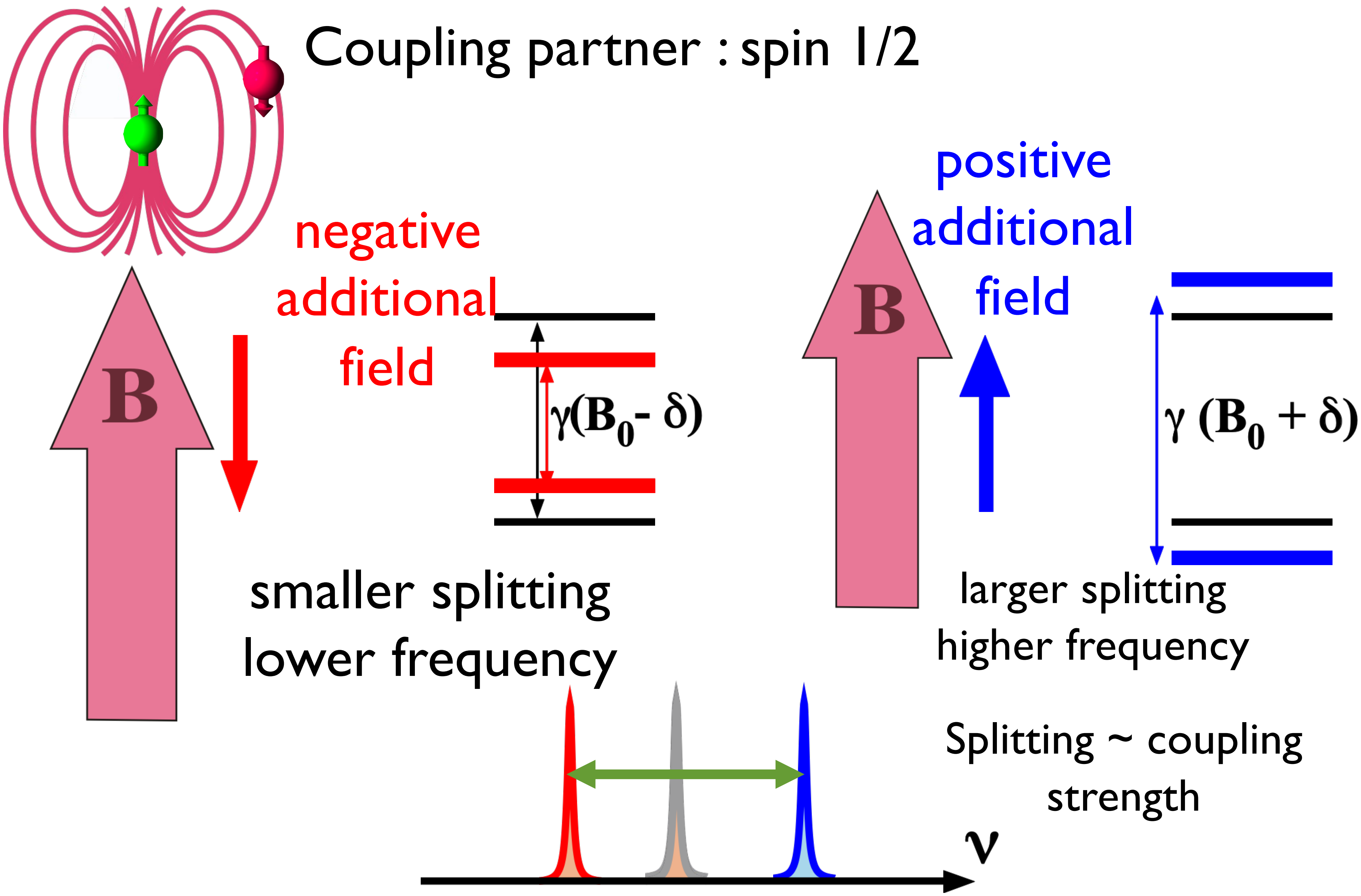
Hard pulses →



Weak pulses →

Selective excitation

Coupled spin systems



Coupling Hamiltonian

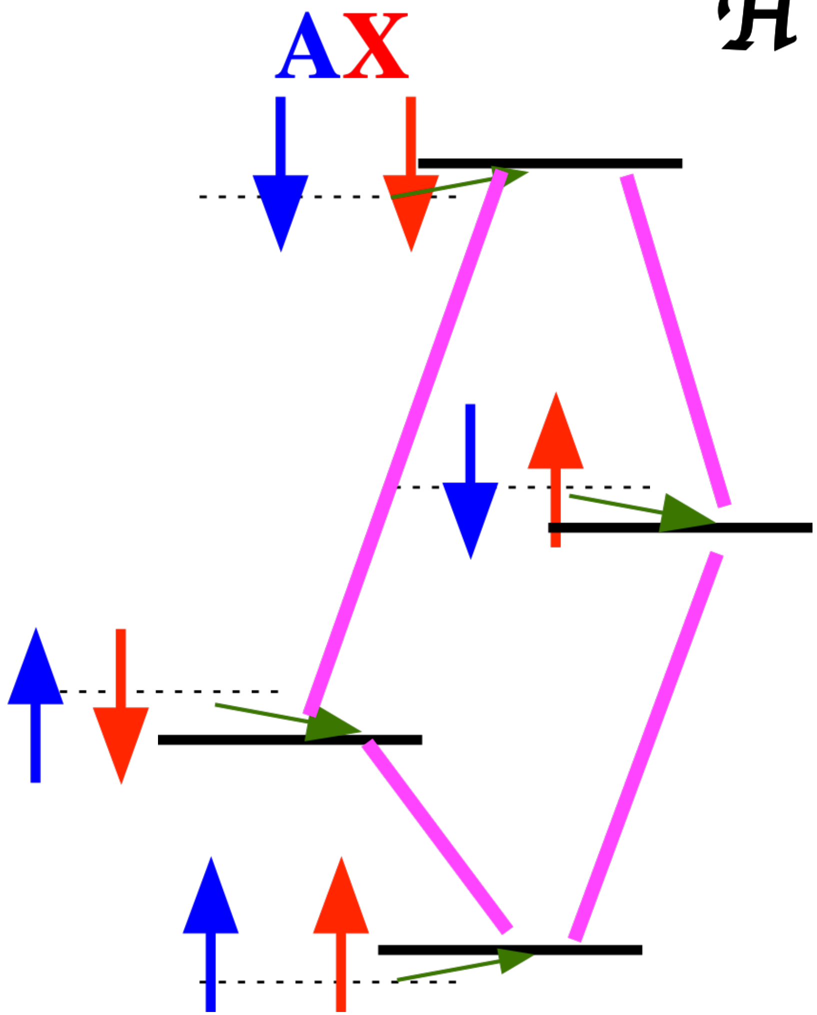
$$\mathcal{H} = \mathcal{H}_z + \mathcal{H}_{AX} = -\omega_A \mathbf{A}_z - \omega_X \mathbf{X}_z + d\mathbf{A}_z\mathbf{X}_z$$

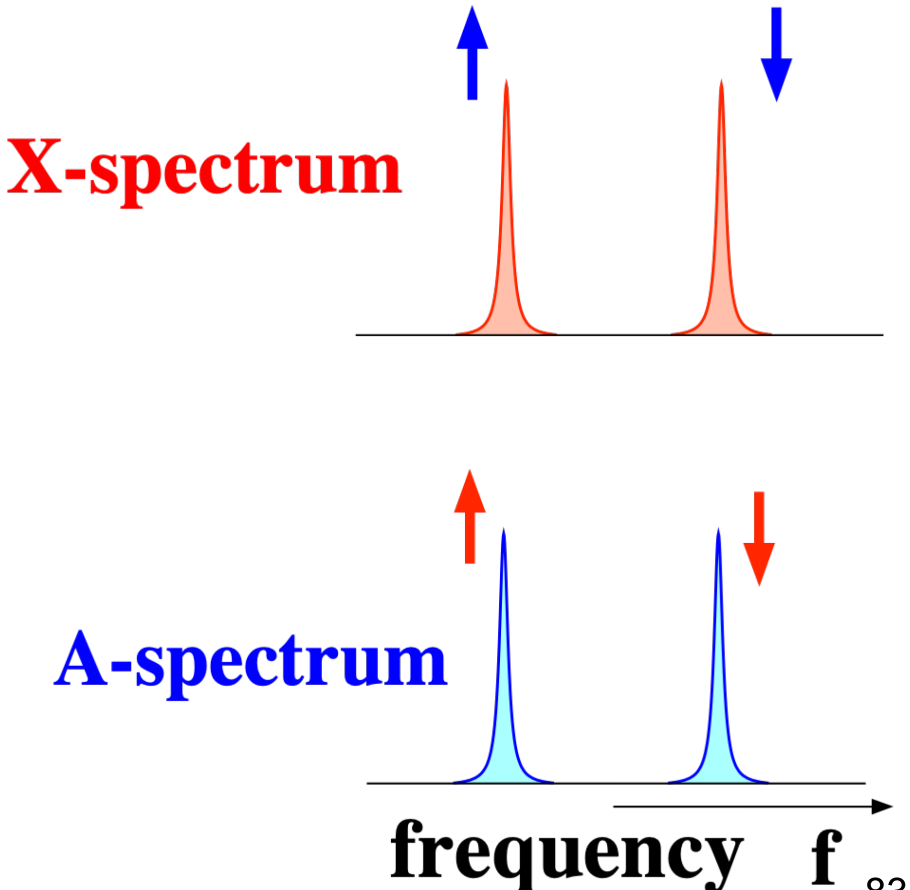
$$= \frac{1}{2}\begin{pmatrix} -\omega_A - \omega_X + \frac{d}{2} & & & \\ & -\omega_A + \omega_X - \frac{d}{2} & & \\ & & \omega_A - \omega_X - \frac{d}{2} & \\ & & & \omega_A + \omega_X + \frac{d}{2} \end{pmatrix}$$

Coupled Spin Systems

Coupled 2-spin system

$$\mathcal{H} = -\omega_A A_z - \omega_X X_z + d\, A_z X_z$$





Pseudopure States

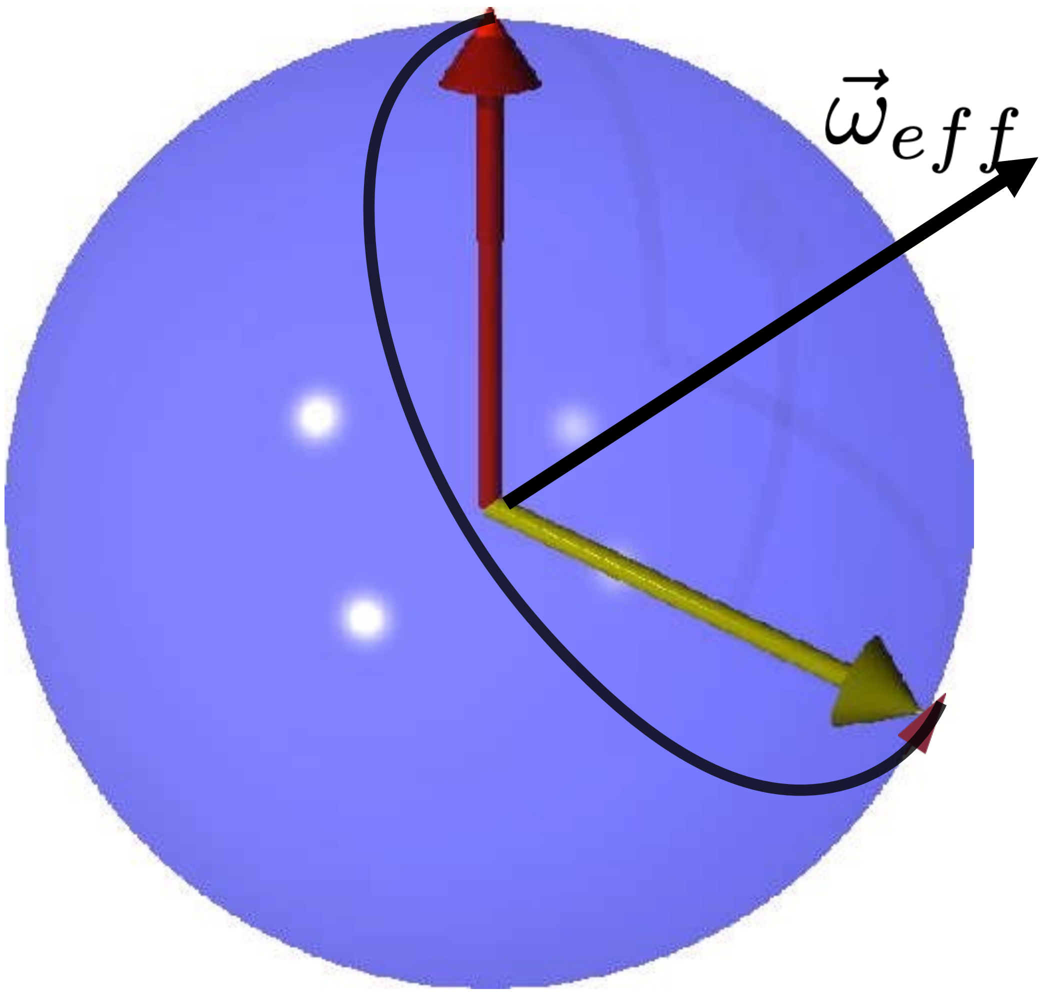
$$\boldsymbol{\rho}_{pp} \propto \beta \mathbf{1} + \alpha \boldsymbol{\rho}_p$$

pure state

- Pseudopure states are (for all practical purposes) equivalent to pure states
- pp state preparation associated with exponential signal loss
- Some algorithms can be applied directly to mixed states

Single Qubit Gates

Time evolution of spins =
Larmor precession around effective field



$$U = e^{-i\mathcal{H}\tau/\hbar}$$
$$= e^{i\vec{\omega}_{eff}\cdot\vec{S}\tau/\hbar}$$

arbitrary rotations are possible by appropriate choice of ω_{eff}, τ

B_1 axis can be changes by RF initial phase

10.2 NMR as a Molecular Quantum Computer

10.2.1 Spins as Qubits

10.2.2 Coupled spin systems

10.2.3 Pseudo / effective pure states

10.2.4 Single-qubit gates

10.2.5 Two-qubit gates

10.2.6 Readout

10.2.7 Readout in multi-qubit systems

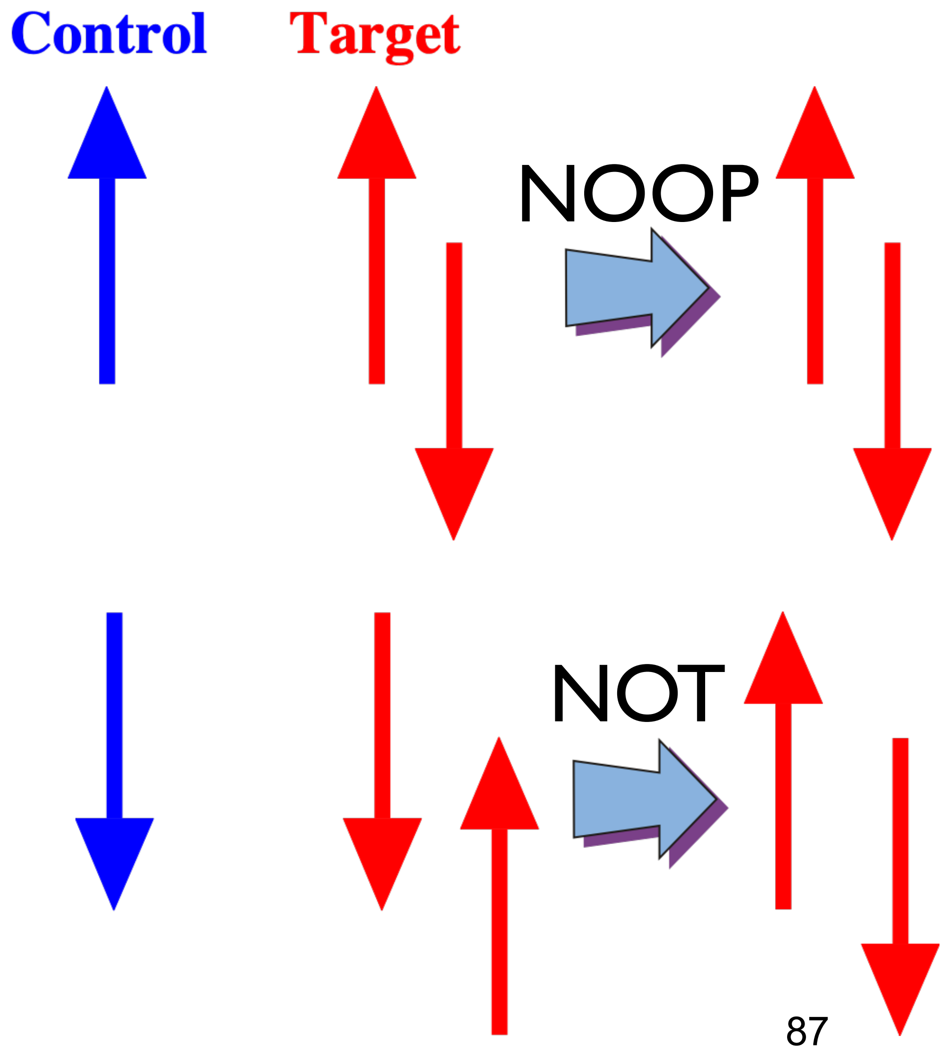
10.2.8 Quantum state tomography

10.2.9 DiVincenzos criteria

Two Qubit Gates

Required for universal quantum computer:
2-qubit gate, e.g. CNOT

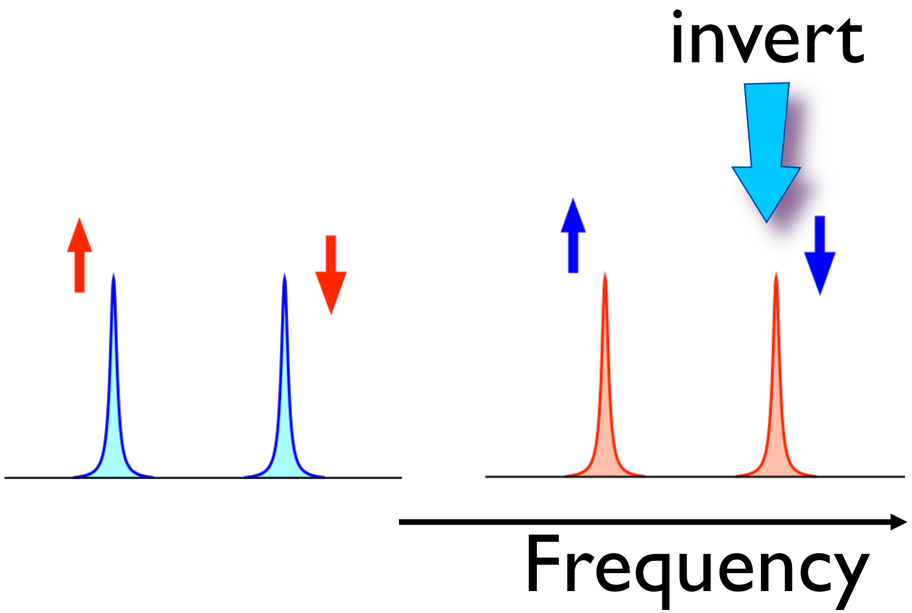
$$\begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

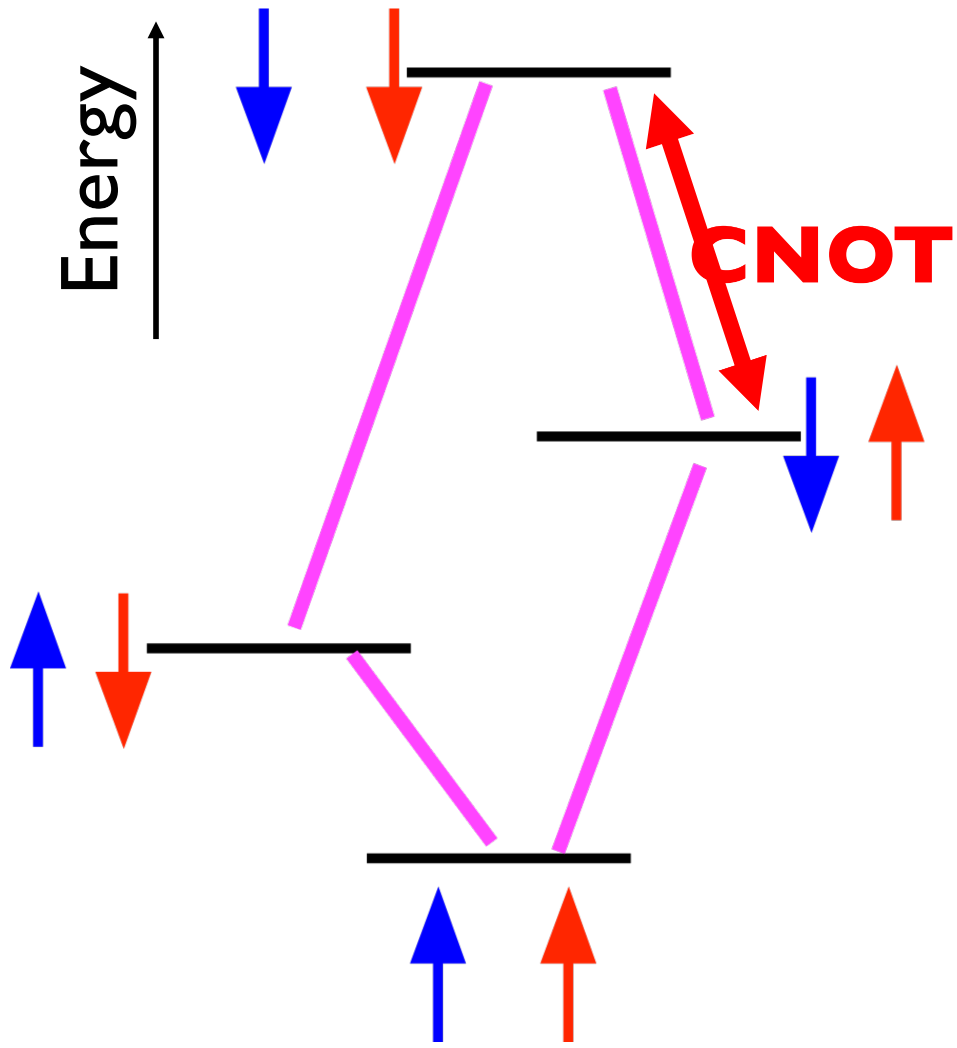


1) Selective Pulses

Example: CNOT

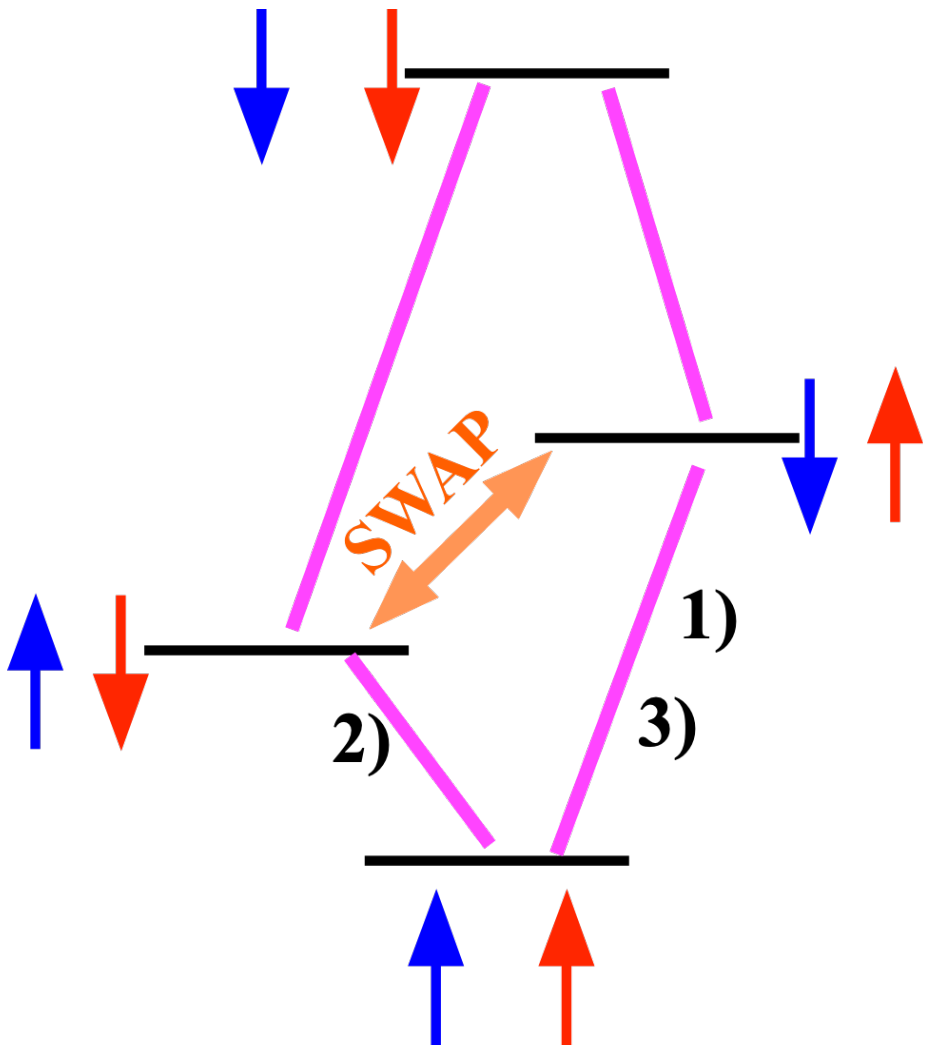
$$\begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

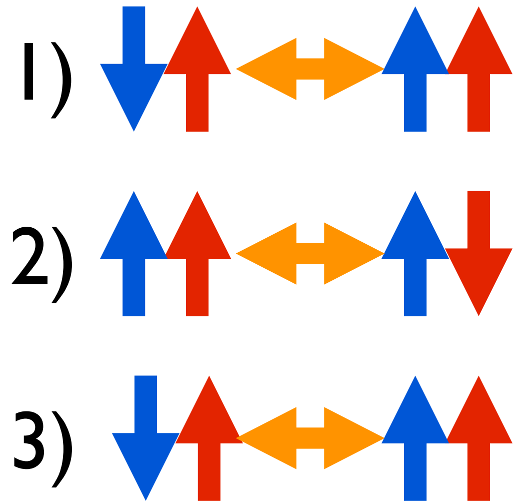




SWAP by Selective Pulses

$$\mathrm{SWAP} = \mathrm{CNOT}_{1\to 2} \cdot \mathrm{CNOT}_{2\to 1} \cdot \mathrm{CNOT}_{1\to 2}.$$

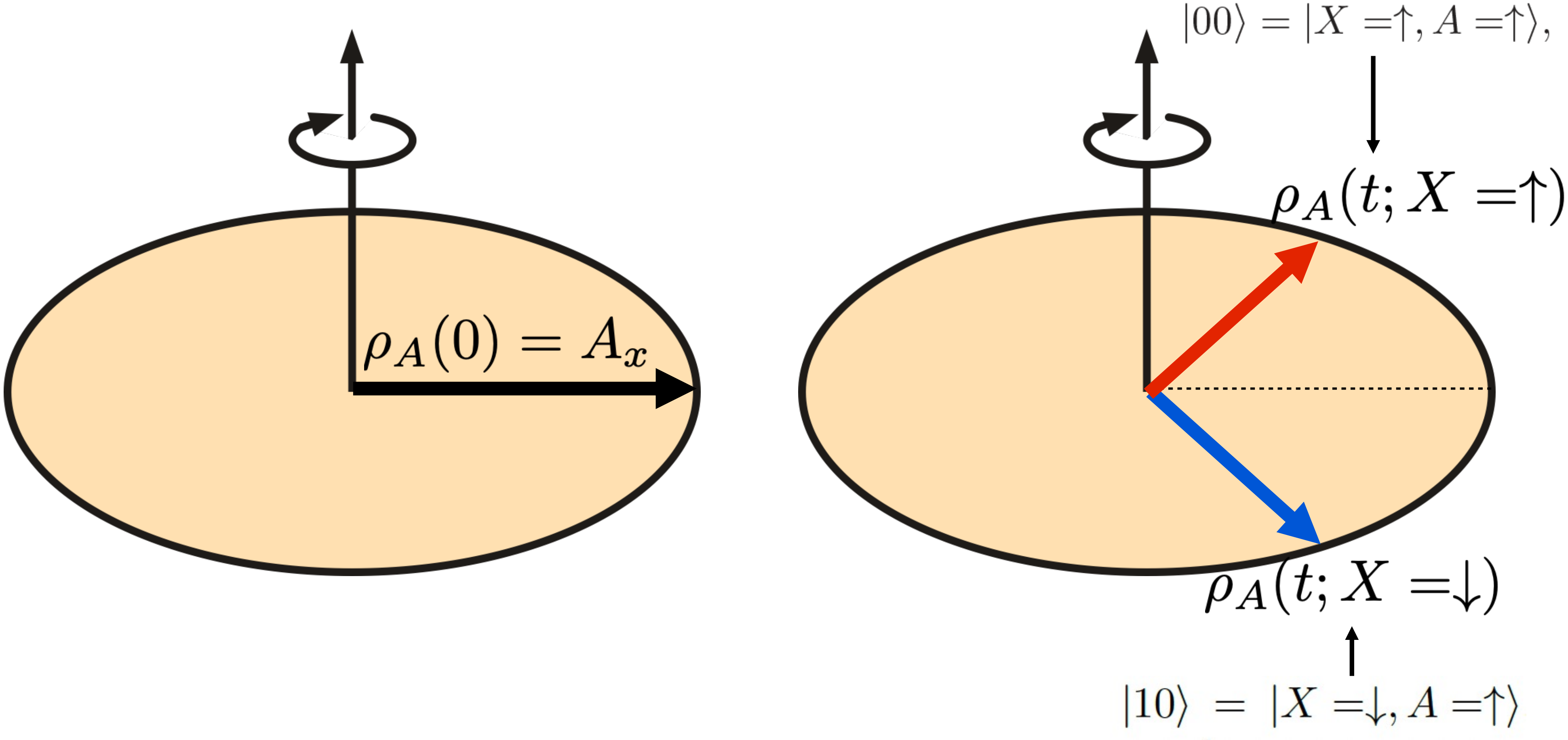




Conditional Dynamics: e.g. Conditional flip

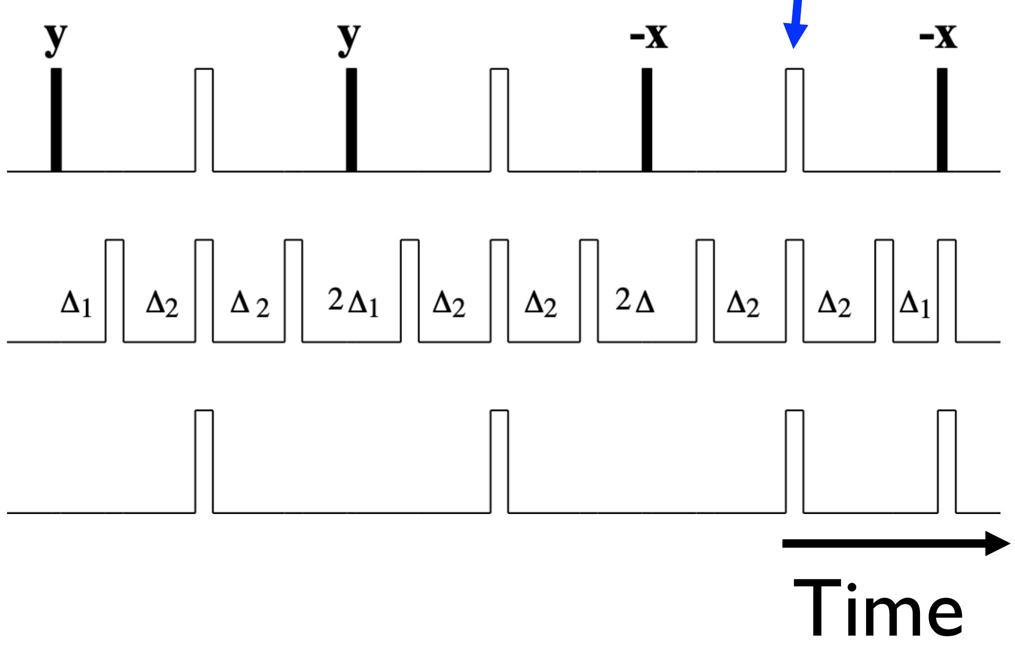
Spin-spin coupling: $\mathcal{H}_{AX} = \frac{d}{\hbar}\mathbf{A}_z\mathbf{X}_z$

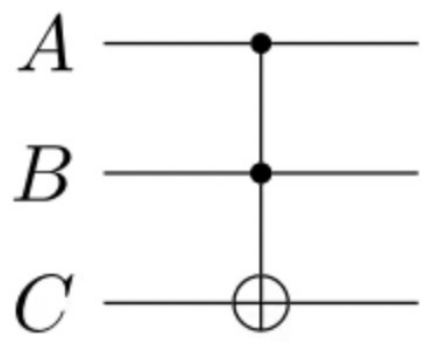
Dynamics of Spin I depends on state of spin S



TOFFOLI Gate or CCNOT

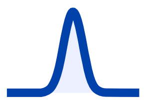
π/2 pulse **π pulse**





Soft pulse
(shaped pulse)

$$|110\rangle \leftrightarrow |111\rangle$$



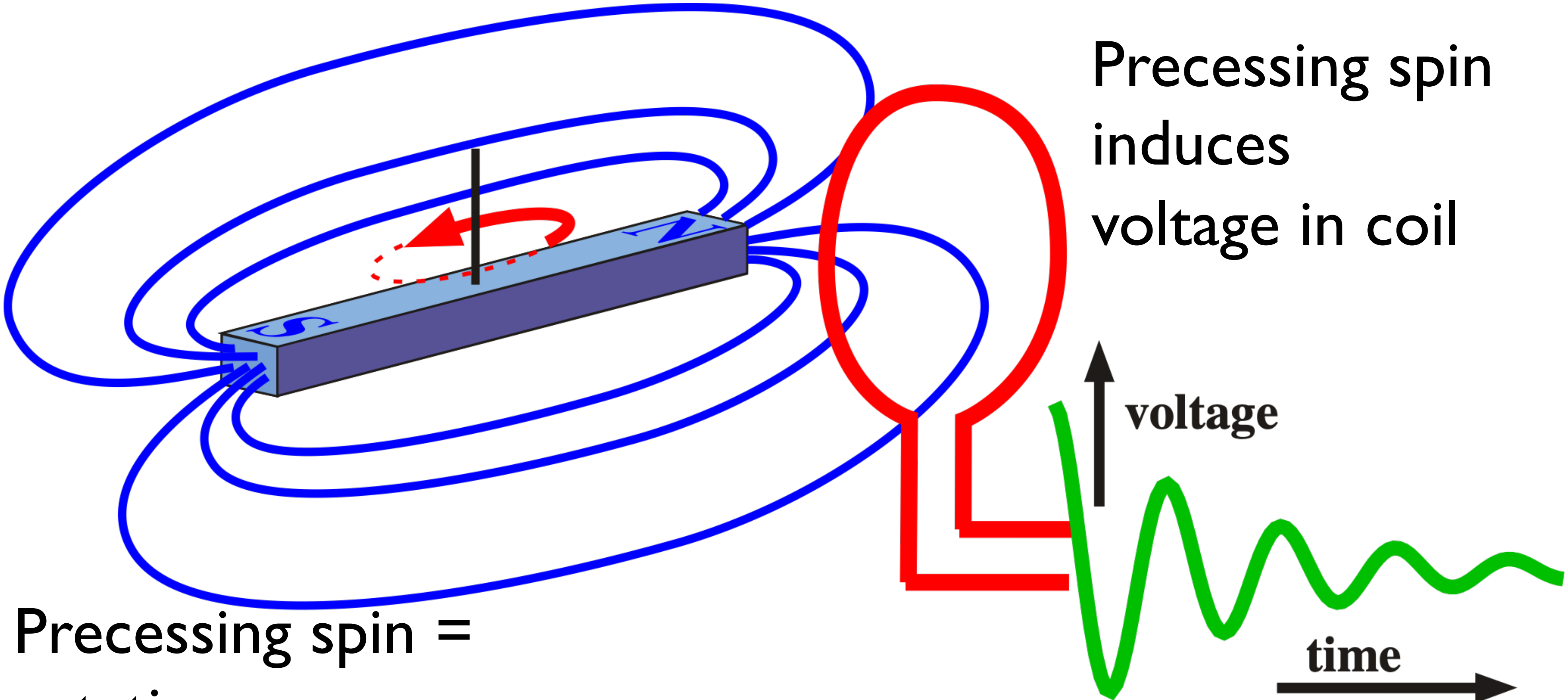
Pulses + free precession

$$\pi_y^{|110\rangle\leftrightarrow|111\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Mahesh, PhD thesis (2003)

Detection

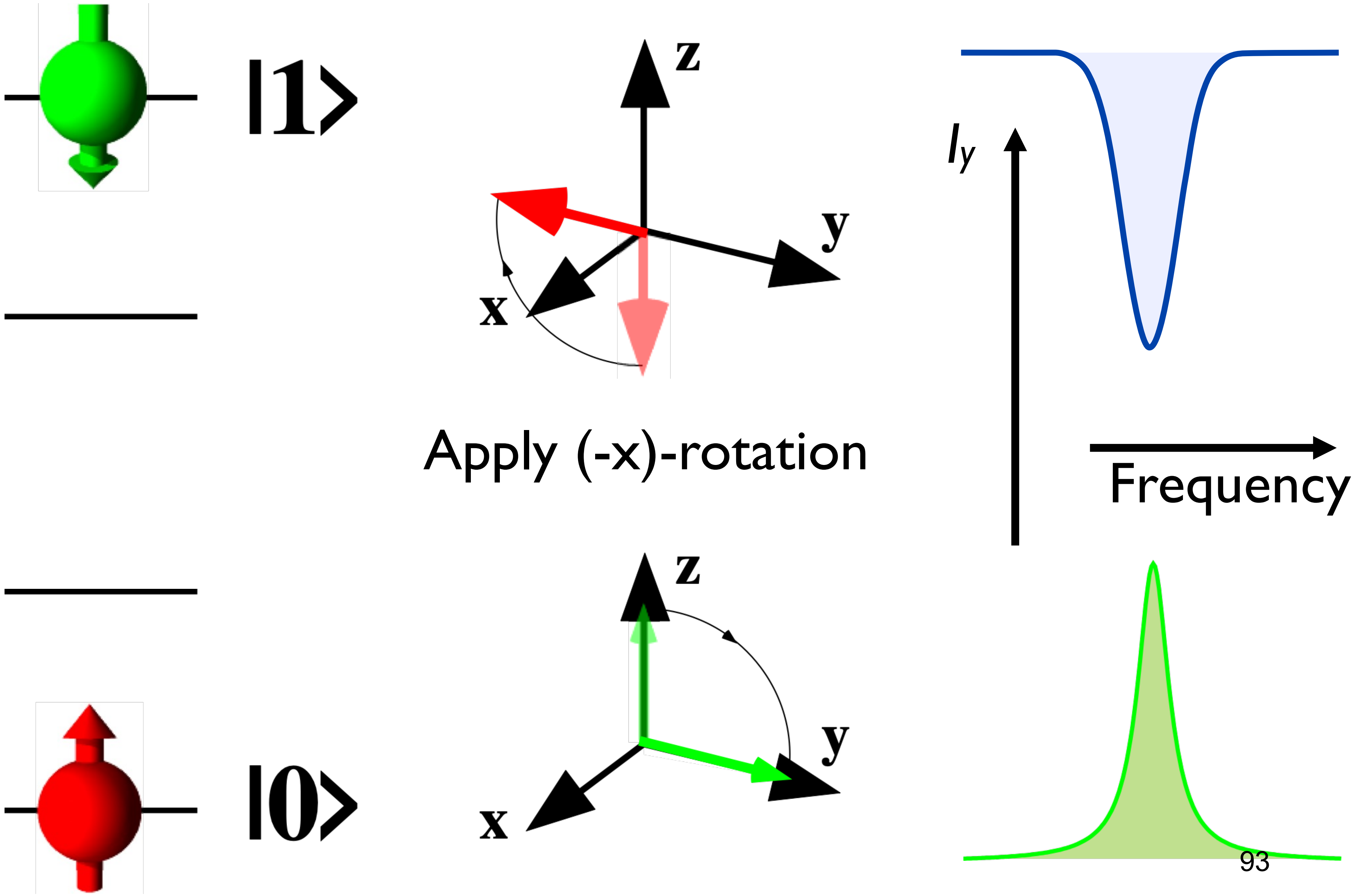
Detection of precessing magnetisation by Faraday effect



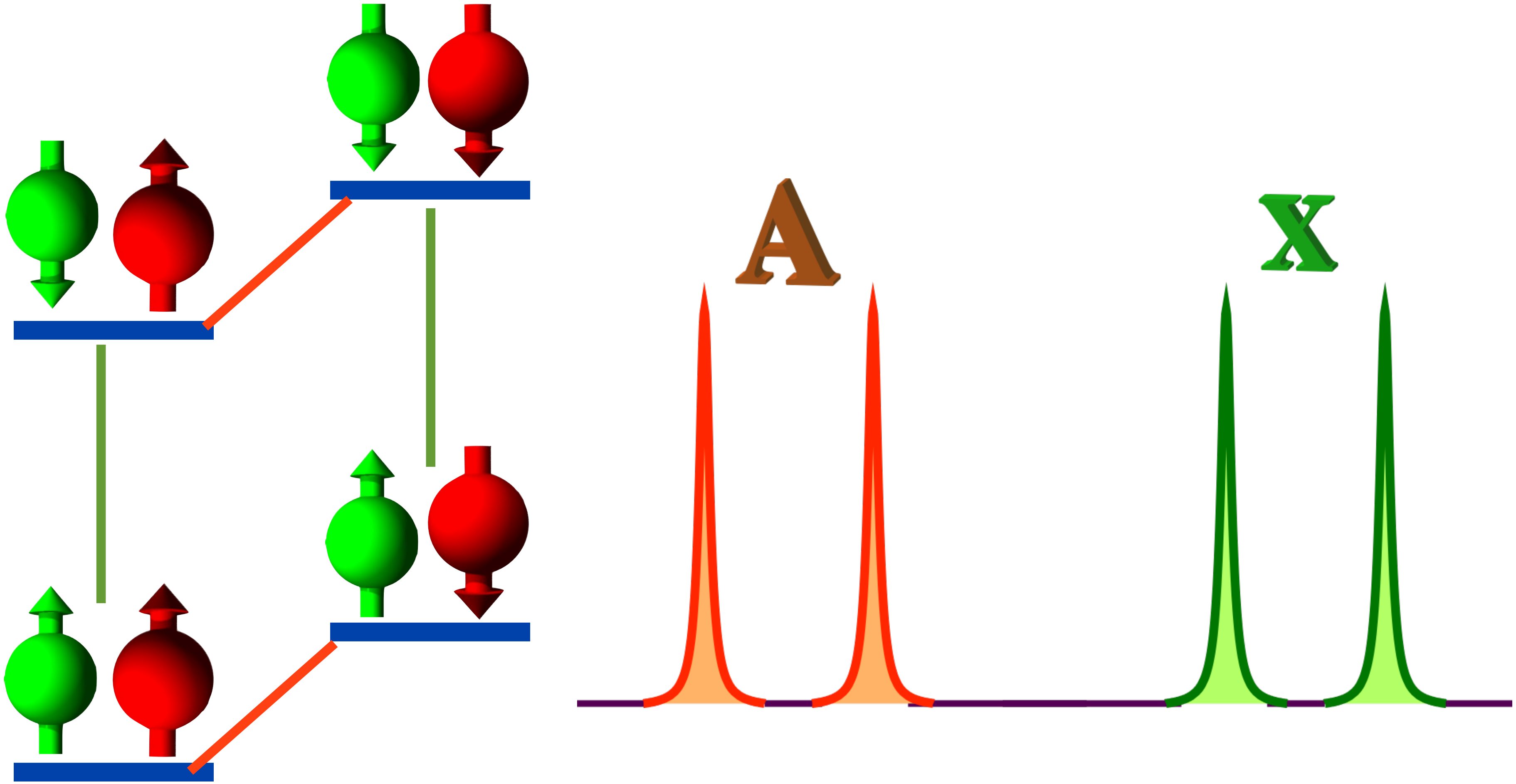
Precessing spin induces voltage in coil

Precessing spin = rotating magnetization

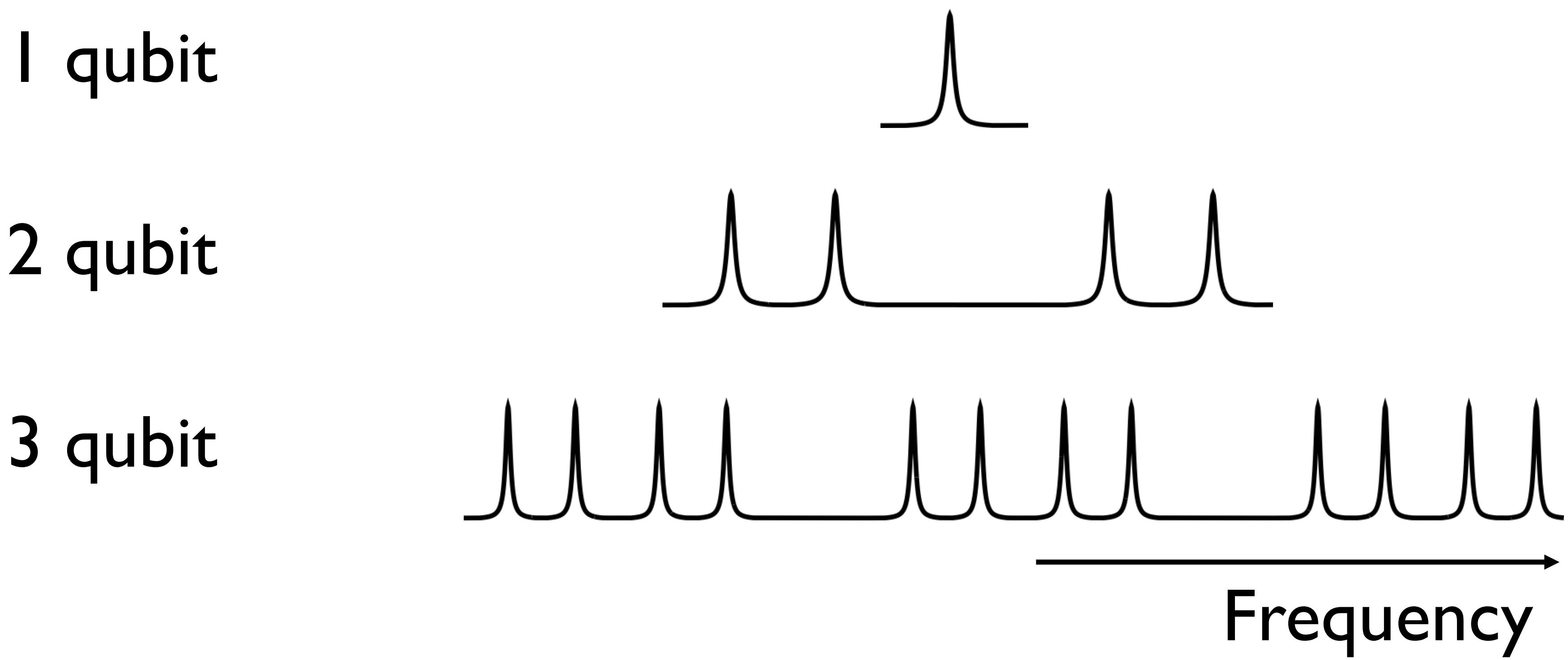
Measuring Populations



AX Detection

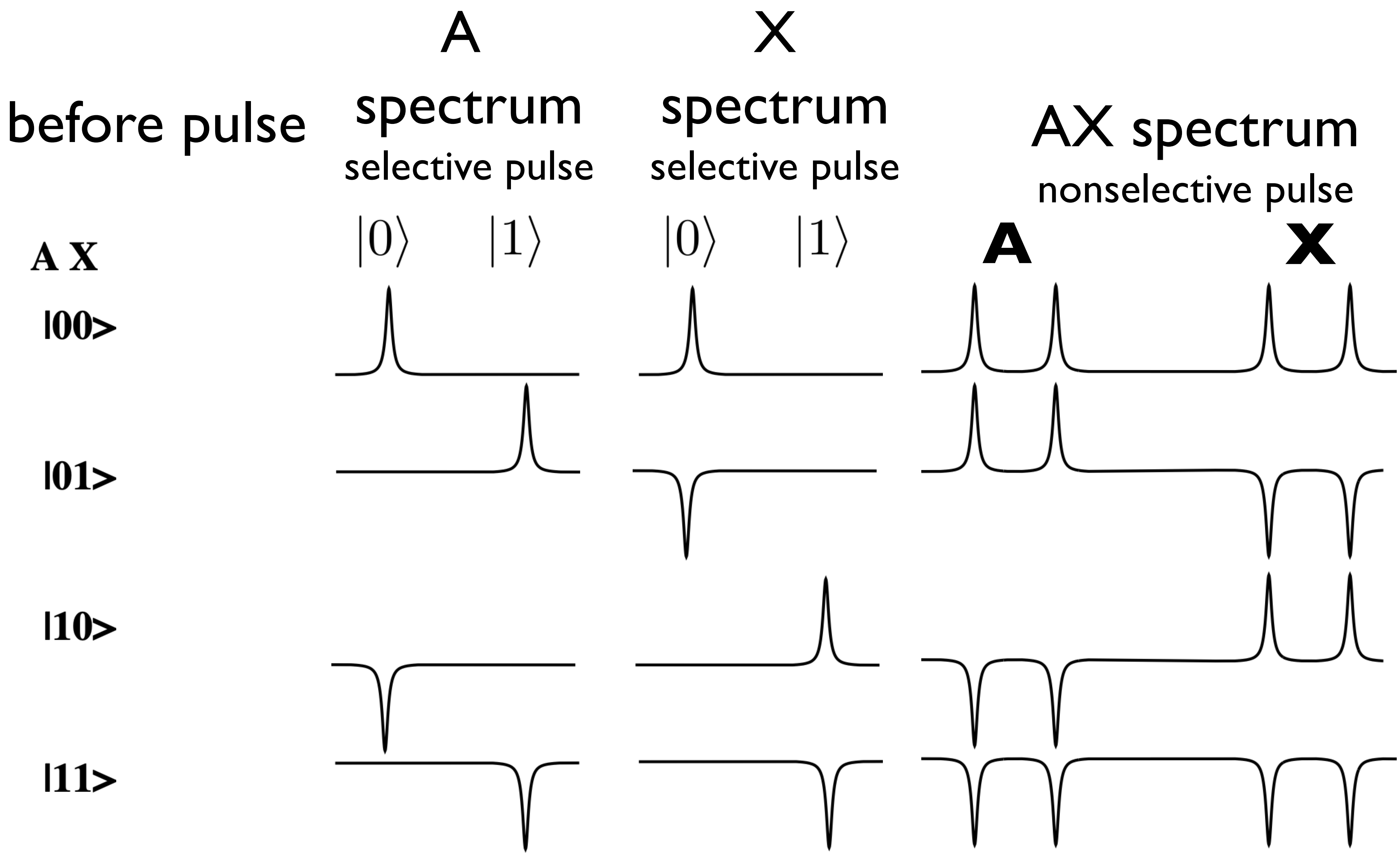


Multi-Spin Systems



total number of lines: $n_L = N2^{N-1}$

Qubit-selective Readout

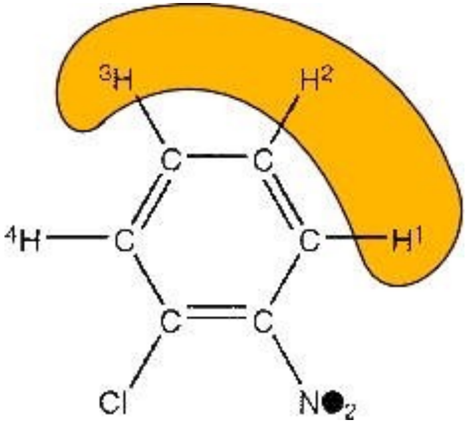


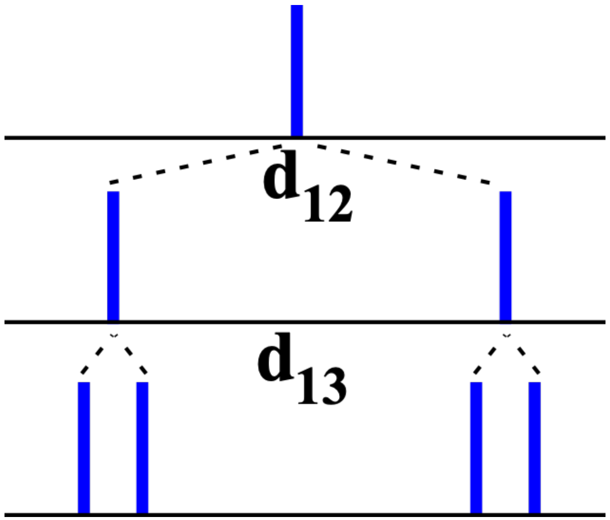
Multi-Qubit Readout

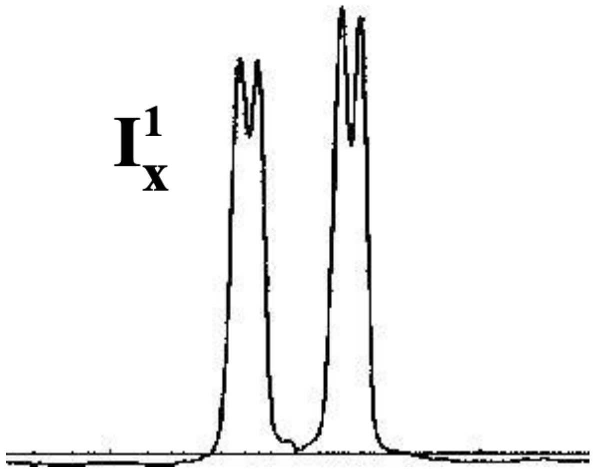
Nuclear magnetic resonance spectroscopy: An experimentally accessible paradigm for quantum computing ?

David G. Cory , Mark D. Price , Timothy F. Havel

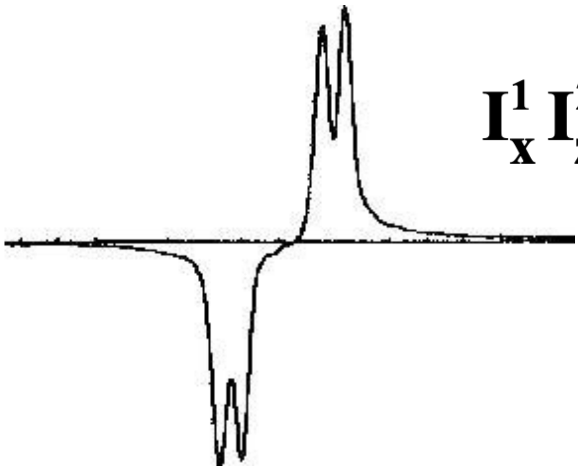
Physica D 120 (1998) 82–101



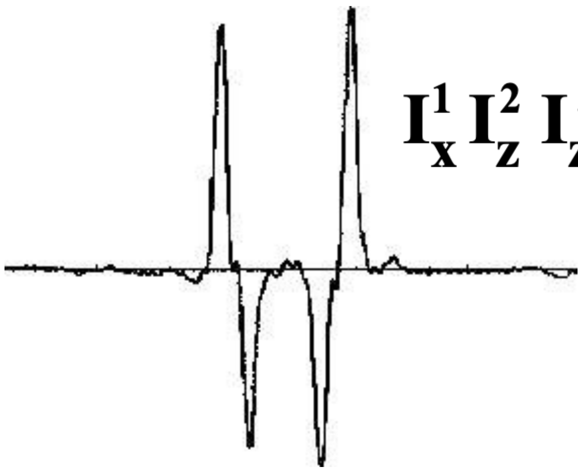




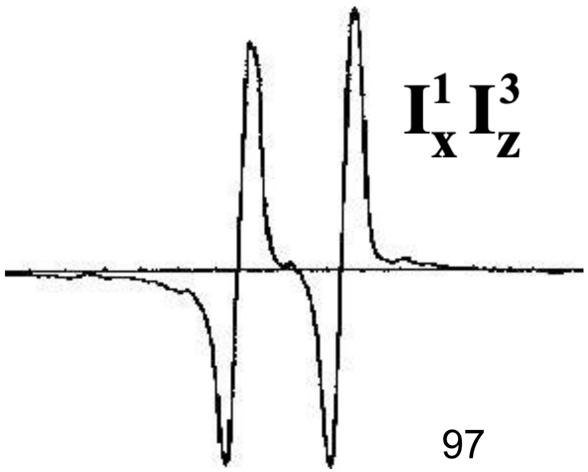
I_x^1



$I_x^1 I_z^2$

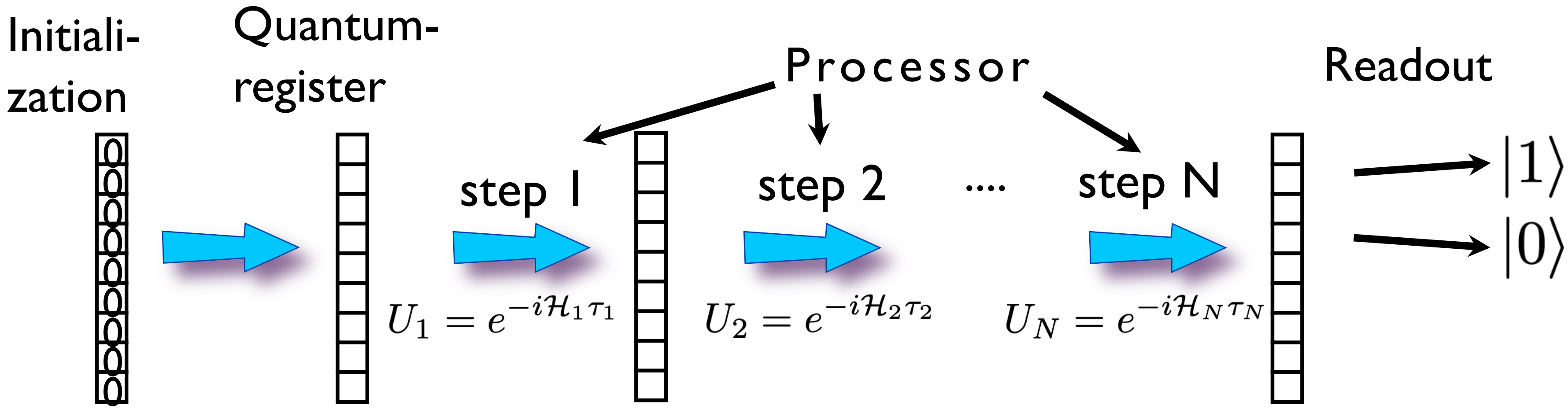


$I_x^1 I_z^2 I_z^3$



$I_x^1 I_z^3$

diVincenzo's Criteria



1) Well characterized qubits, ~~**scalable system**~~

2) Initialization into a well defined state.

3) Long decoherence times.

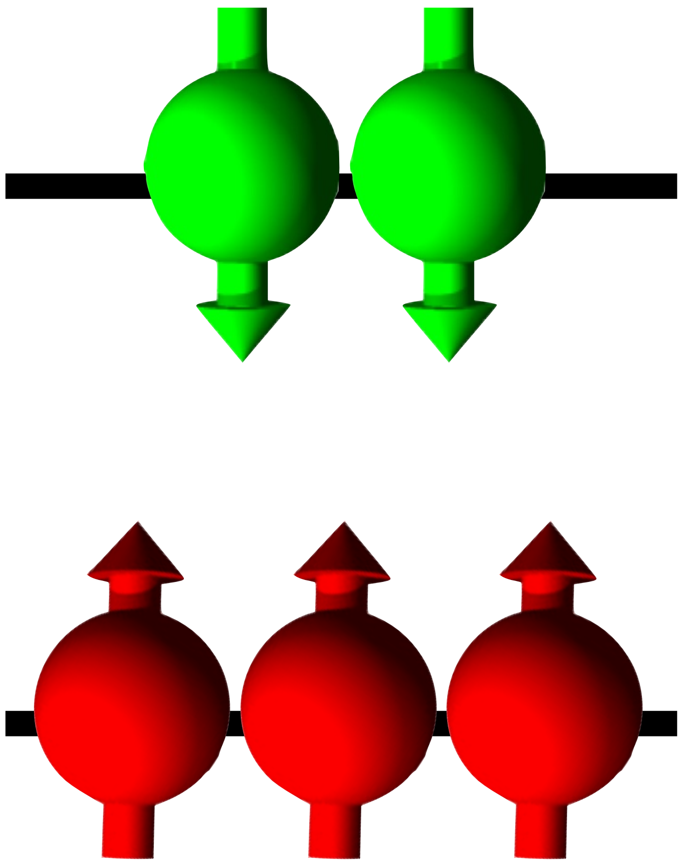
4) Universal set of quantum gates.

5) Qubit-selective readout.

diVincenzo #2

Initialization

Initialization into a well defined state.



Thermal relaxation

$$\rho_{eq} = \mathbb{1} + \underset{\sim 10^{-5}}{\varepsilon} \, \mathbf{I_z}$$

Problems:

- not a pure state
- relaxation is slow

diVincenzo #3

Decoherence

1) **Well characterized qubits, scalable system**

2) **Initialization into a well defined state.**

3) **Long decoherence times.**

4) **Universal set of quantum gates.**

5) **Qubit-selective readout.**

Typical relaxation times ~ 1 s in liquid state NMR
Typical gate duration ~ 10 ms → approx. 100 gates

Gates

1) Well characterized qubits, scalable system

2) Initialization into a well defined state.

3) Long decoherence times.

4) Universal set of quantum gates.

5) Qubit-selective readout.

gate = unitary transformation $\mathbf{U = e^{iHt}}$

diVincenzo #5

Readout

1) Well characterized qubits, scalable system

2) Initialization into a well defined state.

3) Long decoherence times.

4) Universal set of quantum gates.

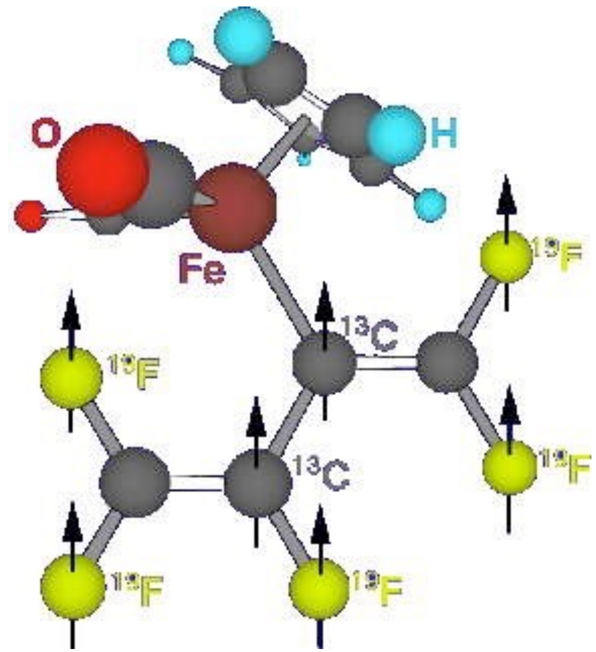
5) Qubit-selective readout.

10.3 NMR Implementation of Shor's Algorithm

Experimental realization of Shor's quantum factoring algorithm using nuclear magnetic resonance

Lieven M. K. Vandersypen*†, Matthias Steffen*†, Gregory Breyta*, Costantino S. Yannoni*, Mark H. Sherwood* & Isaac L. Chuang*†

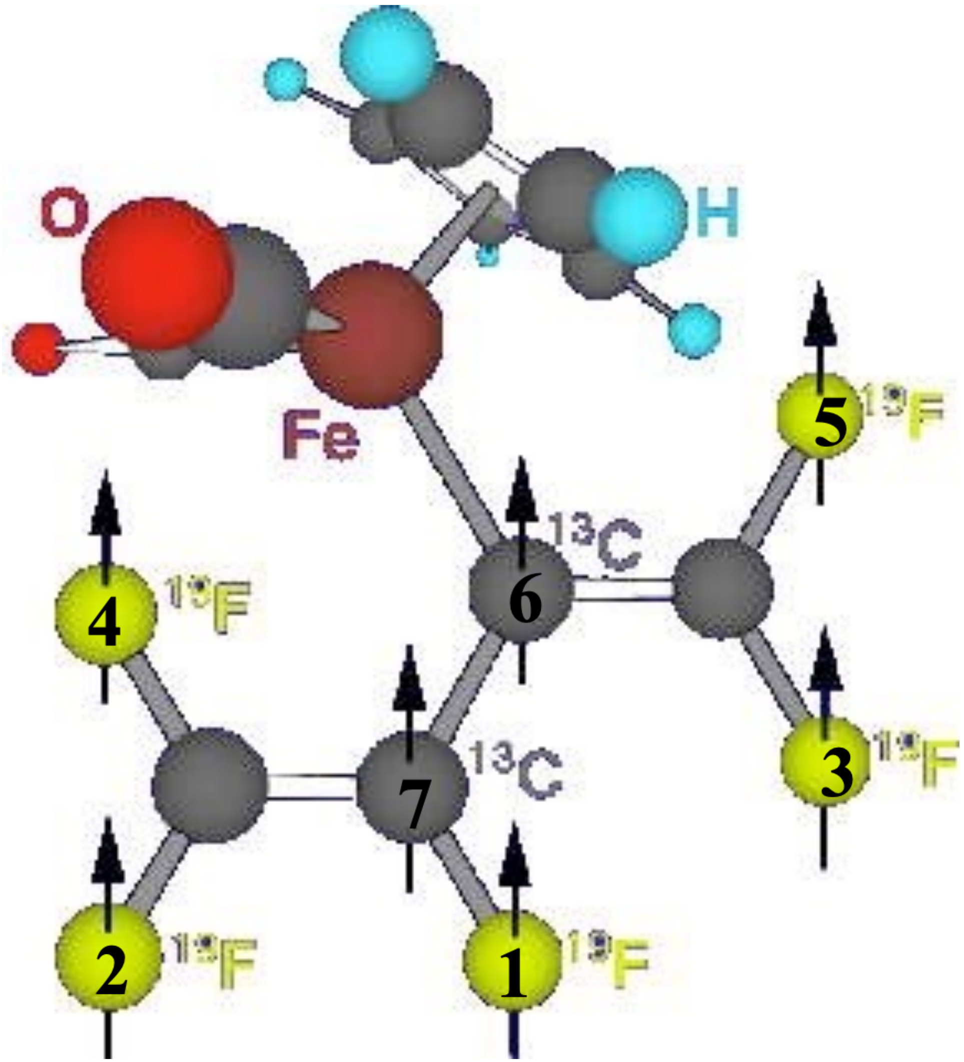
* *IBM Almaden Research Center, San Jose, California 95120, USA*
† *Solid State and Photonics Laboratory, Stanford University, Stanford, California 94305-4075, USA*



Nature 414, 883 (2001).

10.3.1 Qubit implementation
10.3.2 Initialization
10.3.3 Computation
10.3.4 Readout
10.3.5 Decoherence

Quantum Register



Frequencies, relaxation times

	i	$\omega_i/2\pi$	$T_{1,i}$	$T_{2,i}$
^{19}F	1	−22052.0	5.0	1.3
	2	489.5	13.7	1.8
	3	25088.3	3.0	2.5
	4	−4918.7	10.0	1.7
	5	15186.6	2.8	1.8
^{13}C	6	−4519.1	45.4	2.0
	7	4244.3	31.6	2.0

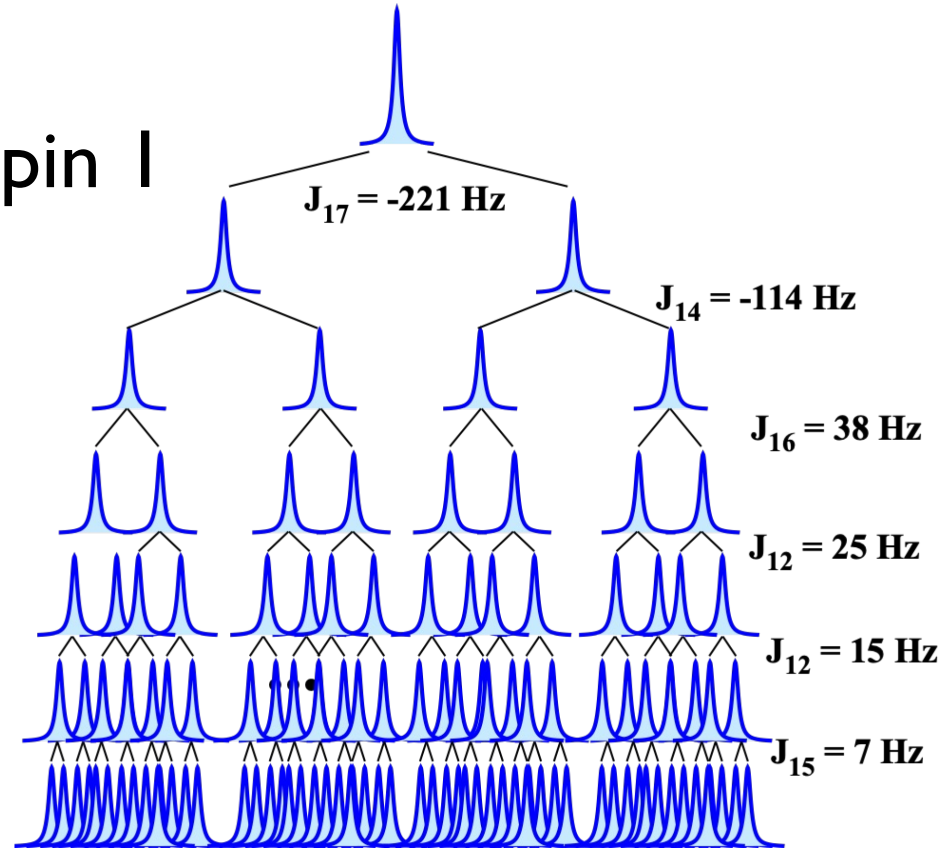
Coupling constants

i	J_{7i}	J_{6i}	J_{5i}	J_{4i}	J_{3i}	J_{2i}
1	−221.0	37.7	6.6	−114.3	14.5	25.16
2	18.6	−3.9	2.5	79.9	3.9	
3	1.0	−13.5	41.6	12.9		
4	54.1	−5.7	2.1			
5	19.4	59.5				
6	68.9					
7						

Resonance frequencies @ 14 T:
^{13}C → 151 MHz, ^{19}F → 565 MHz

Spectrum

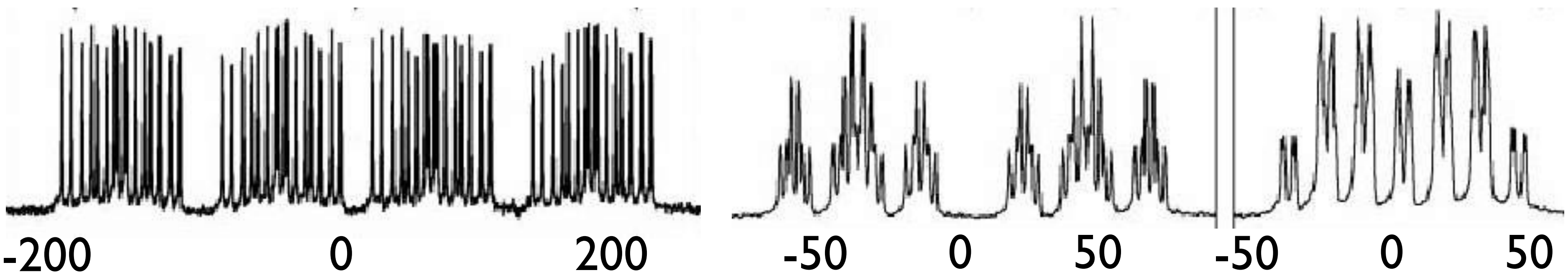
Spin 1



i	J_{7i}	J_{6i}	J_{5i}	J_{4i}	J_{3i}	J_{2i}
1	−221.0	37.7	6.6	−114.3	14.5	25.16
2	18.6	−3.9	2.5	79.9	3.9	
3	1.0	−13.5	41.6	12.9		

$$2^6 = 64 \text{ resonance lines}$$

Spin 2 Spin 3



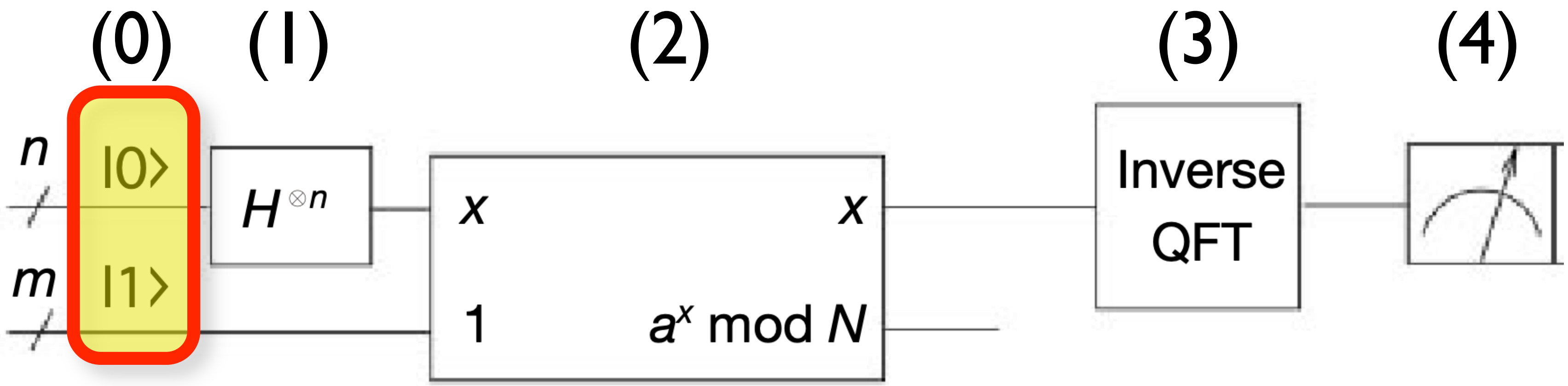
NMR frequency [Hz]

Algorithm: factoring 15

$N = 15$ $\quad$ m must be at least 4 $\longrightarrow$ $2^4 = 16 > 15$

n in the general case 8 $\quad$ $\boxed{N^2 \leq Q := 2^K \leq 2N^2}$

$\longrightarrow$ reduced to 2 $\longrightarrow$ chose $n = 3$, to find additional periods.

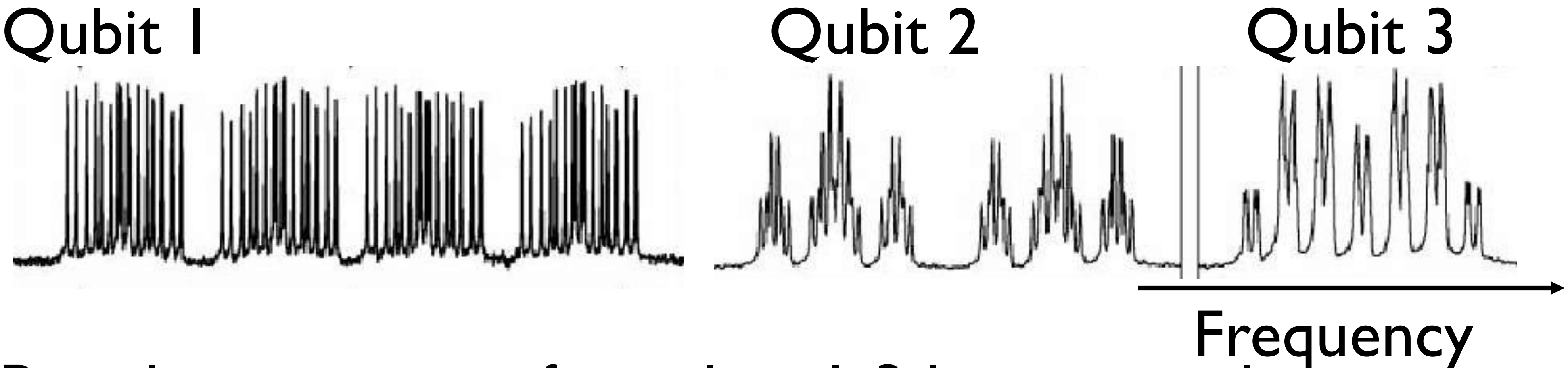


QFT

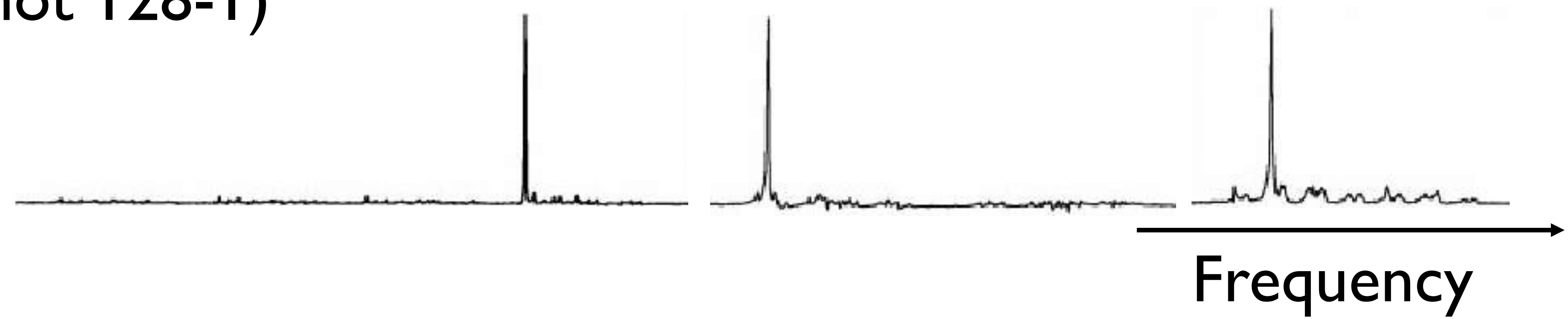
pseudopure state

Initialization

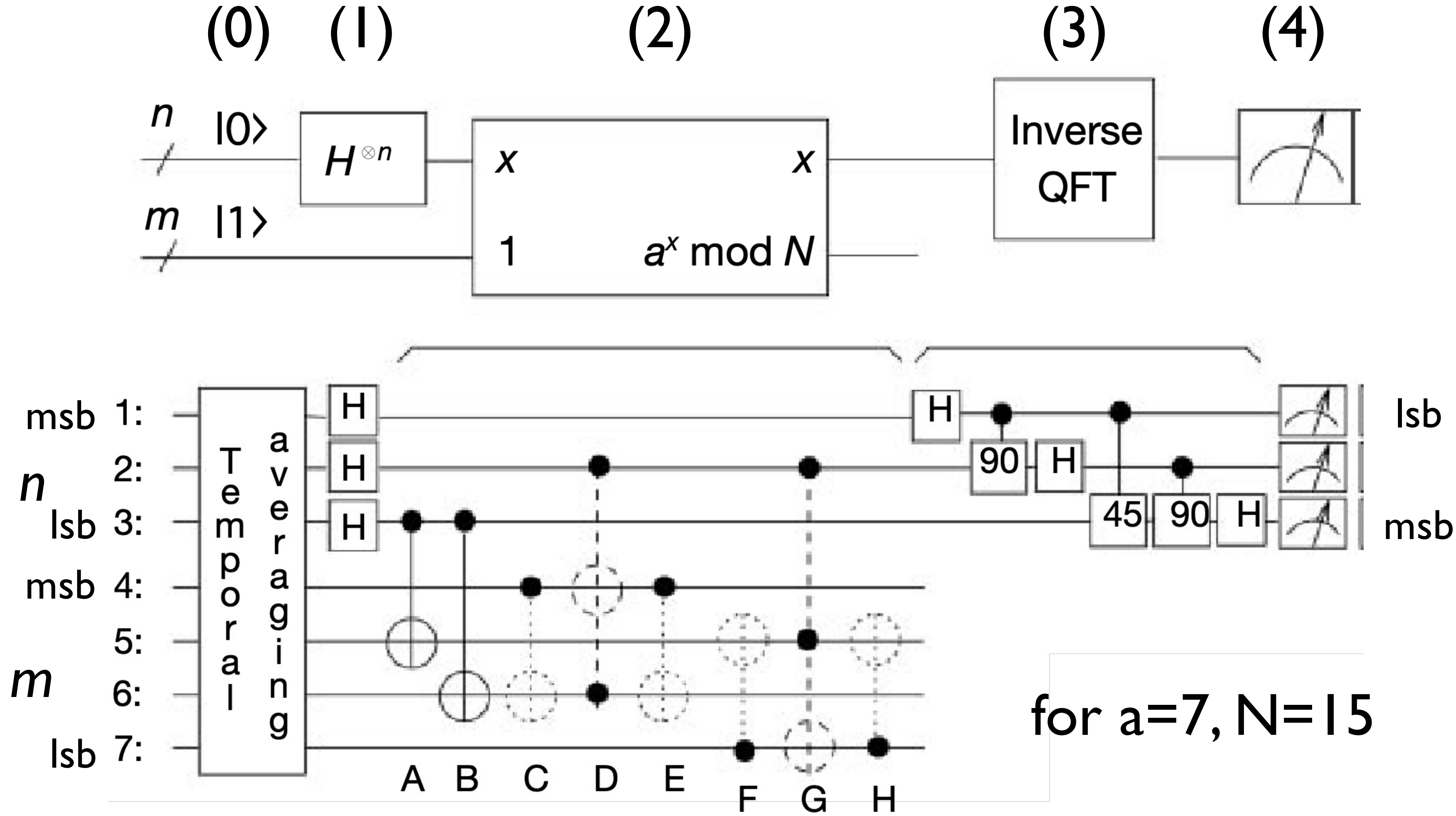
Spectrum taken from thermal equilibrium



Pseudo-pure states for qubits 1-3 by temporal averaging over 36 experiments using symetries, (if not 128-1)



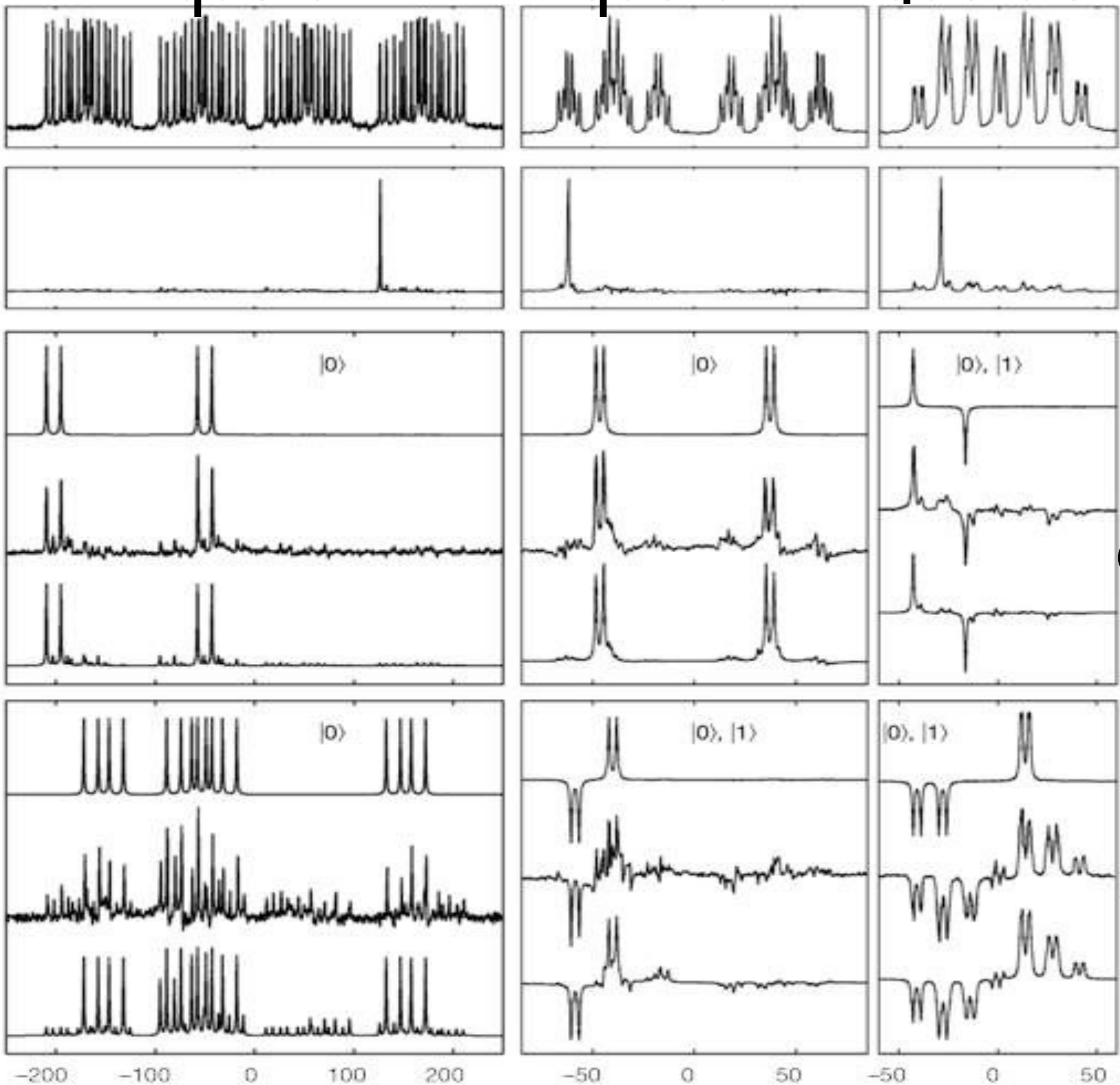
Adapted Algorithm



for a=7, N=15

Experimental Results

qubit 1 qubit 2 qubit 3



Equilibrium spectrum

PPS spectrum

Result for a=11

ideal

experimental

model

Result for a=7

ideal

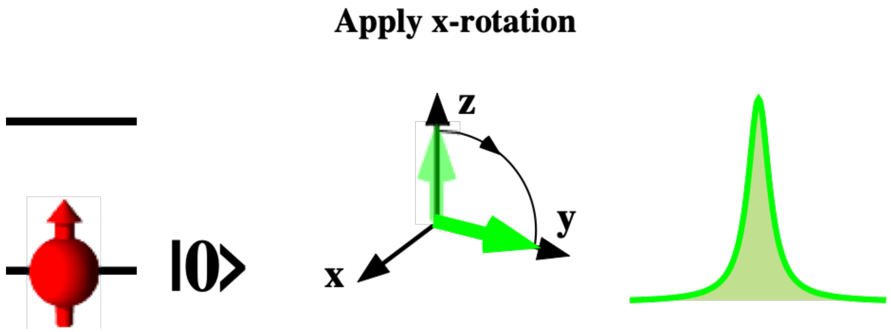
experimental

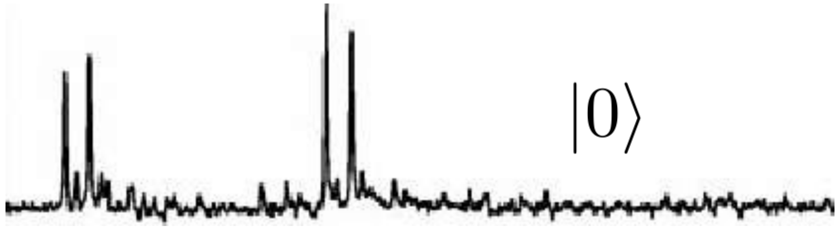
model

Frequency [Hz]

Results for a=11

Readout:
measure populations



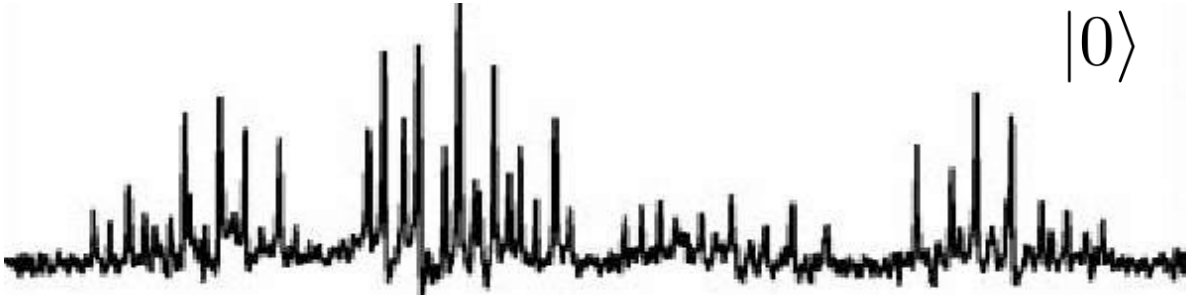
Qubit 1  $|0\rangle$

Qubit 2  $|0\rangle$

Qubit 3  $|0\rangle, |1\rangle$

Results for a=7

Qubit 1

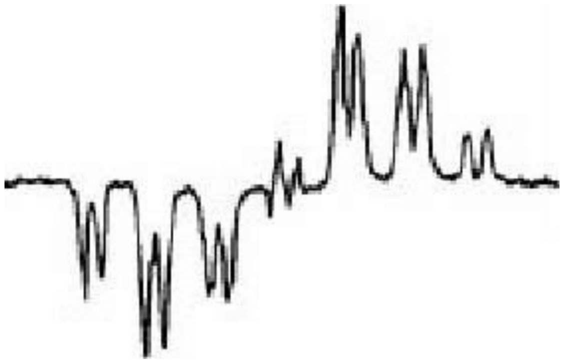


$|0\rangle$

Qubit 2

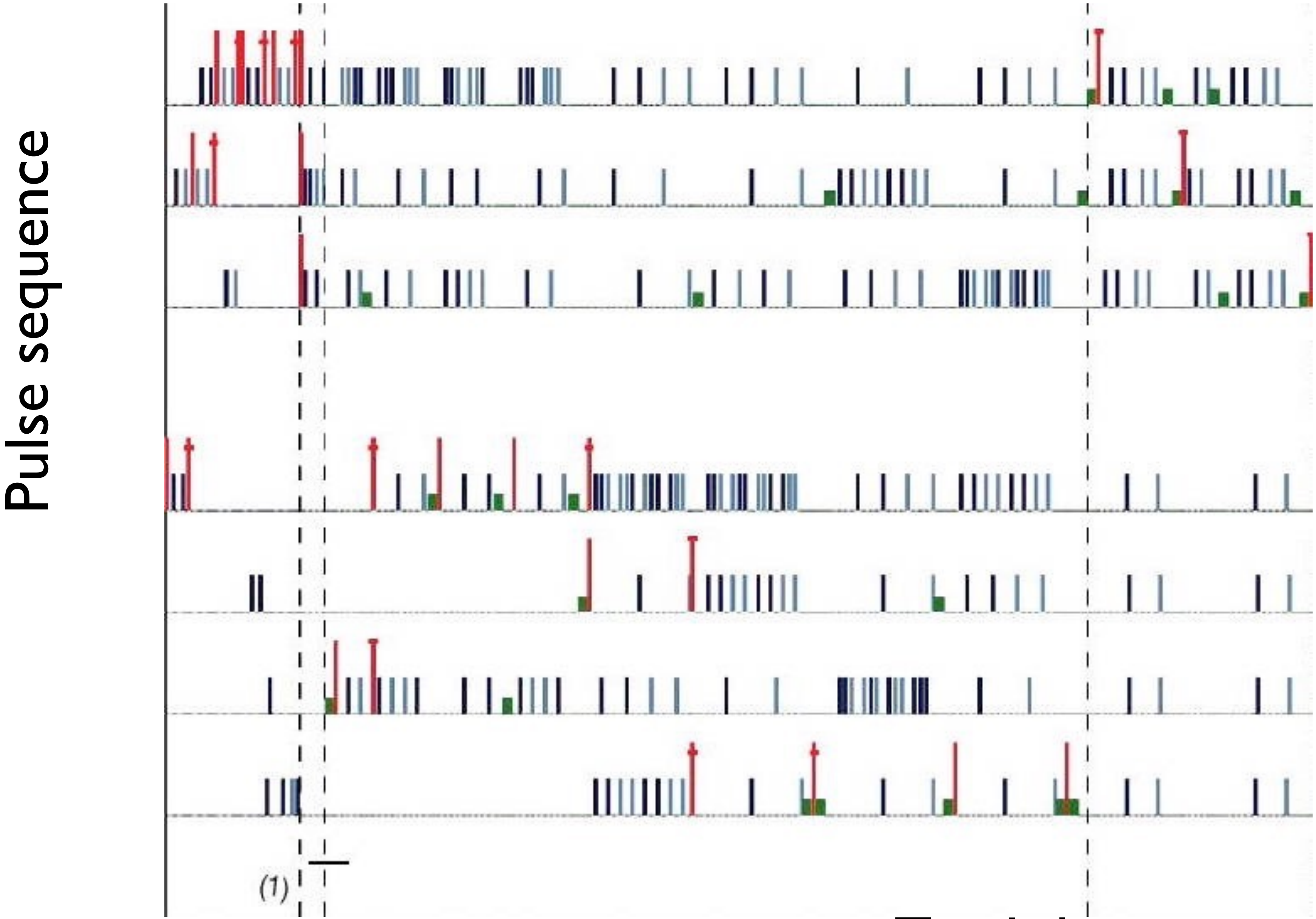
$|0\rangle, |1\rangle$

Qubit 3



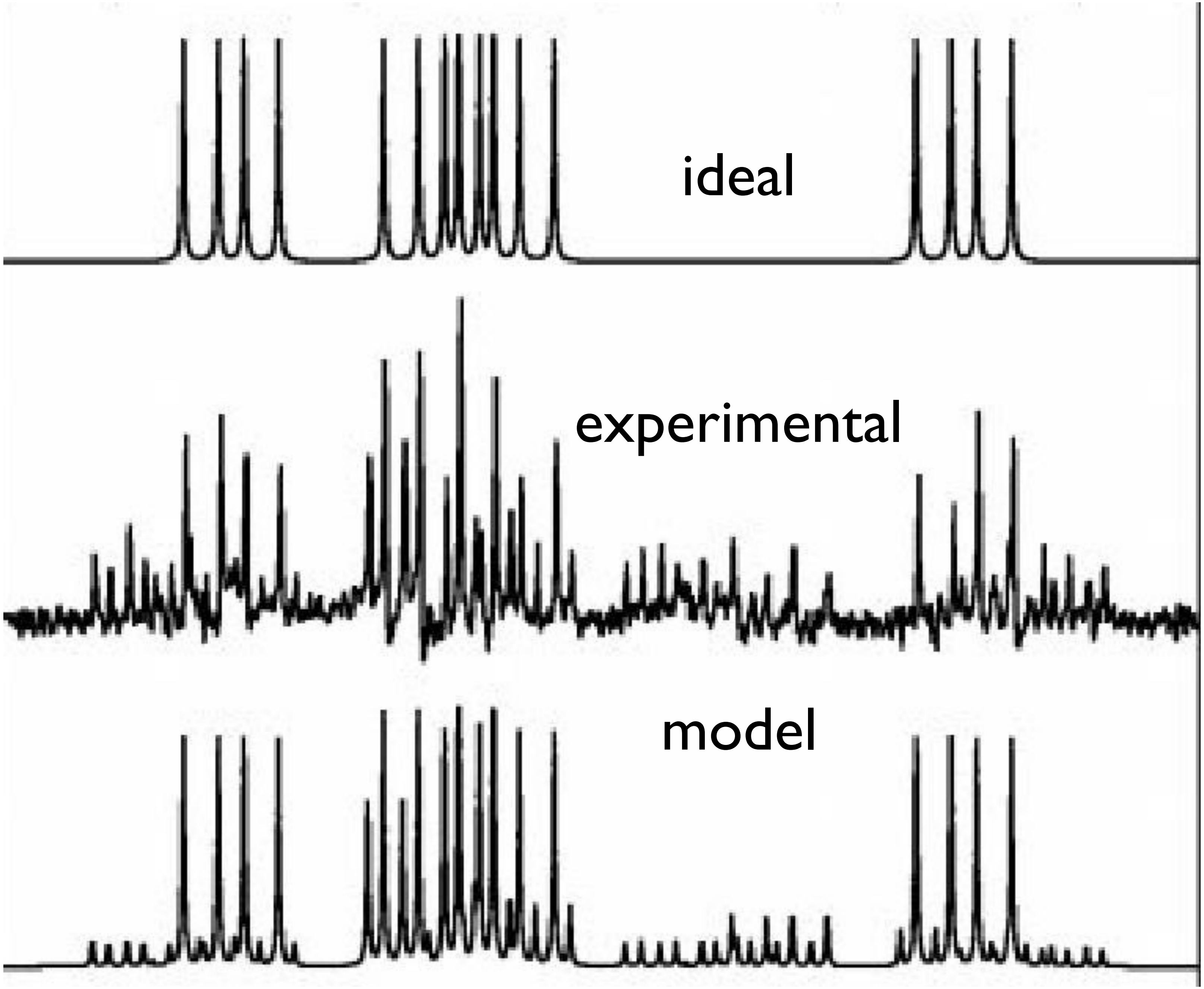
$|0\rangle, |1\rangle$

Decoherence



Total duration ~ 0.7 s

Decoherence Model



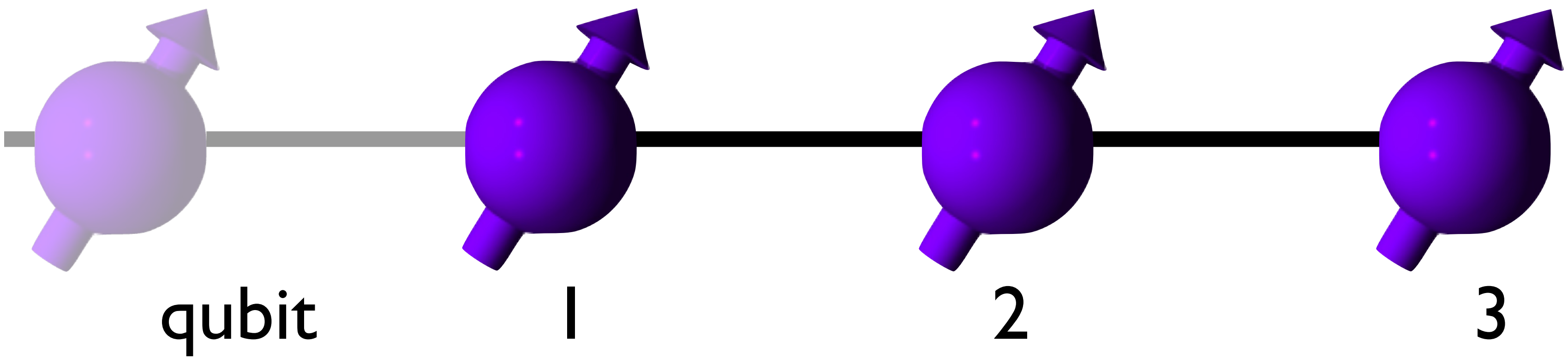
Quantum Computing

Dieter Suter
Gonzalo A. Álvarez, Analía Zwick

Contents

Contact:
Gonzalo A. Alvarez: gonzalo.alvarez@conicet.gov.ar
gonzalo.a.alvarez@gmail.com
Analia Zwick: analia.zwick@gmail.com
Web-page: https://www.famaf.unc.edu.ar/~galvarez/

Spin Chains



State transfer in spin chains

11 April 1997

CHEMICAL PHYSICS LETTERS

ELSEVIER

Chemical Physics Letters 268 (1997) 300–305

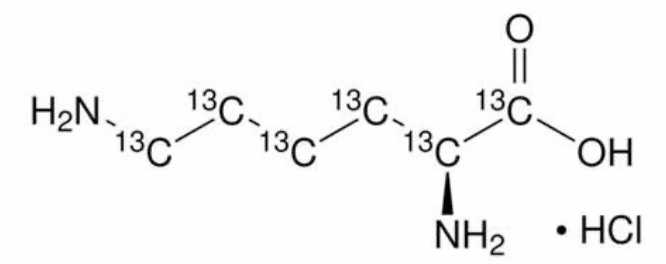
Time-resolved observation of spin waves in a linear chain of nuclear spins

Z.L. Mádi, B. Brutscher, T. Schulte-Herbrüggen, R. Brüschweiler, R.R. Ernst

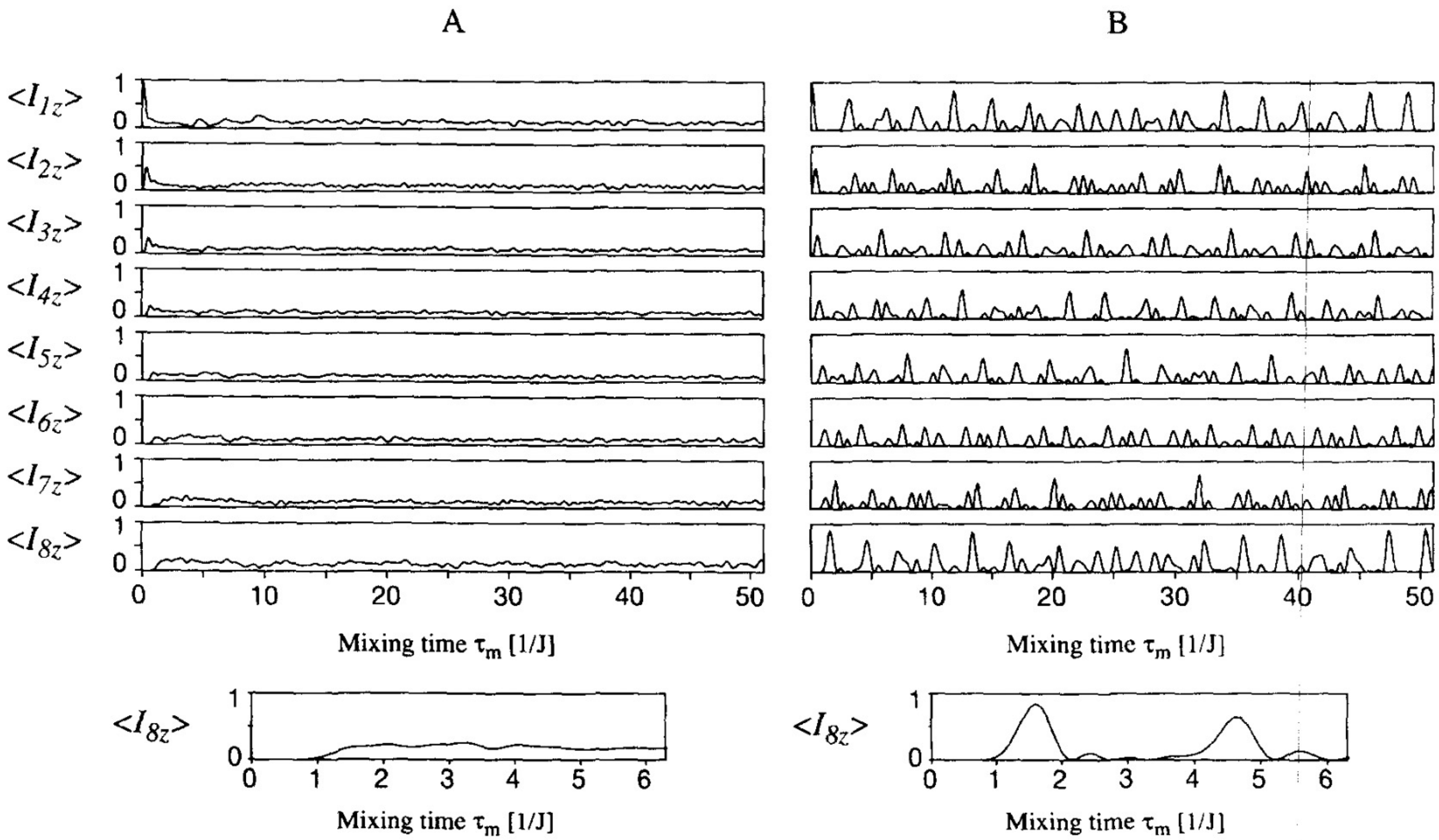
Laboratorium für Physikalische Chemie, ETH Zentrum, 8092 Zürich, Switzerland

Received 30 January 1997; in final form 11 February 1997

fully ^{13}C-labelled lysine



State transfer in spin chains



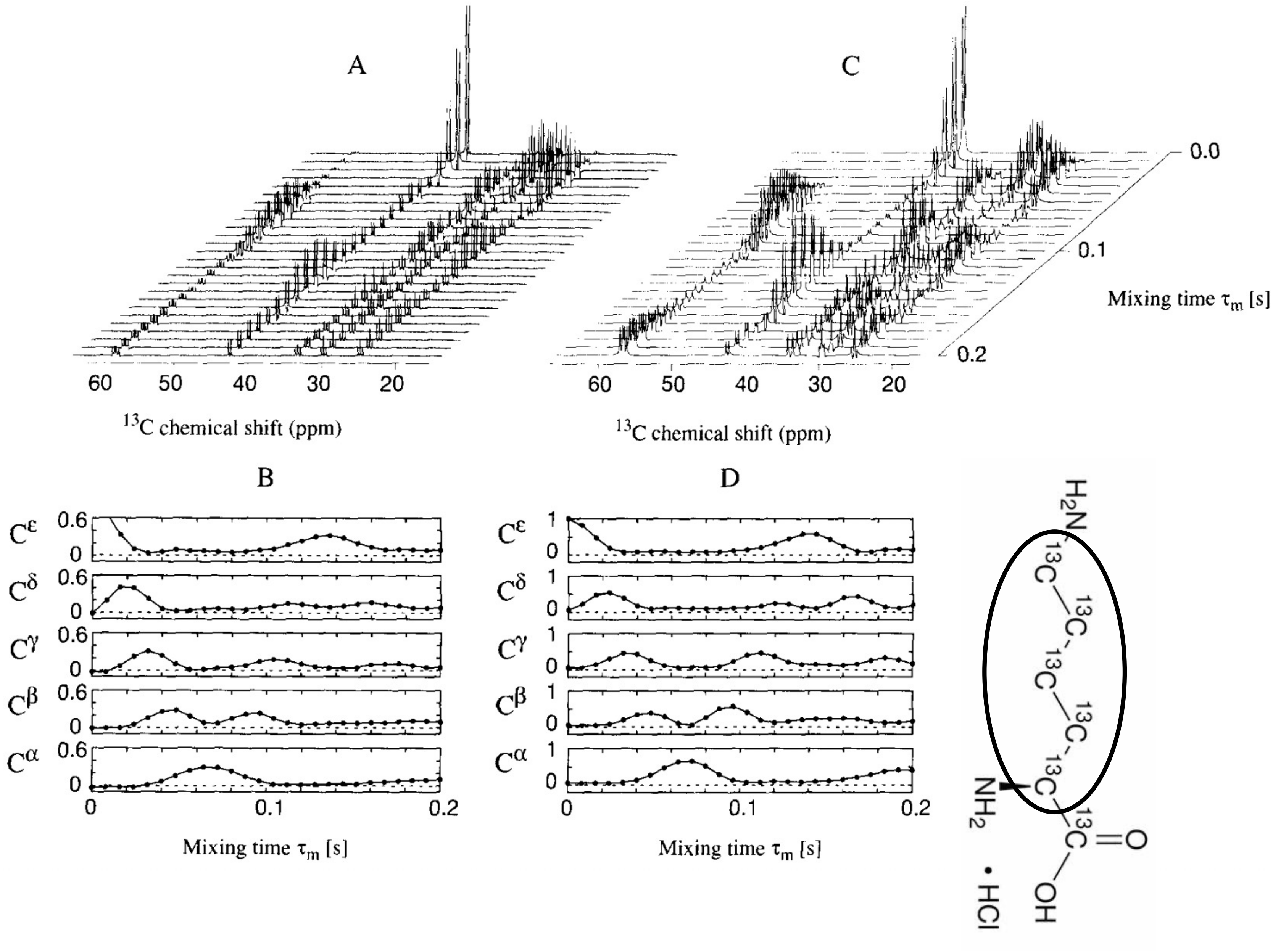
$$H_J^{\mathrm{iso}} = 2\pi \sum_{i<j} J_{ij} \boldsymbol{I}_i \boldsymbol{I}_j$$

$$= 2\pi \sum_{i<j} J_{ij} \{ I_{ix} I_{jx} + I_{iy} I_{jy} + I_{iz} I_{jz} \}$$

$$H_J^{\mathrm{ZQ}} = 2\pi \sum_{i<j} J_{ij} \{ I_{ix} I_{jx} + I_{iy} I_{jy} \}$$

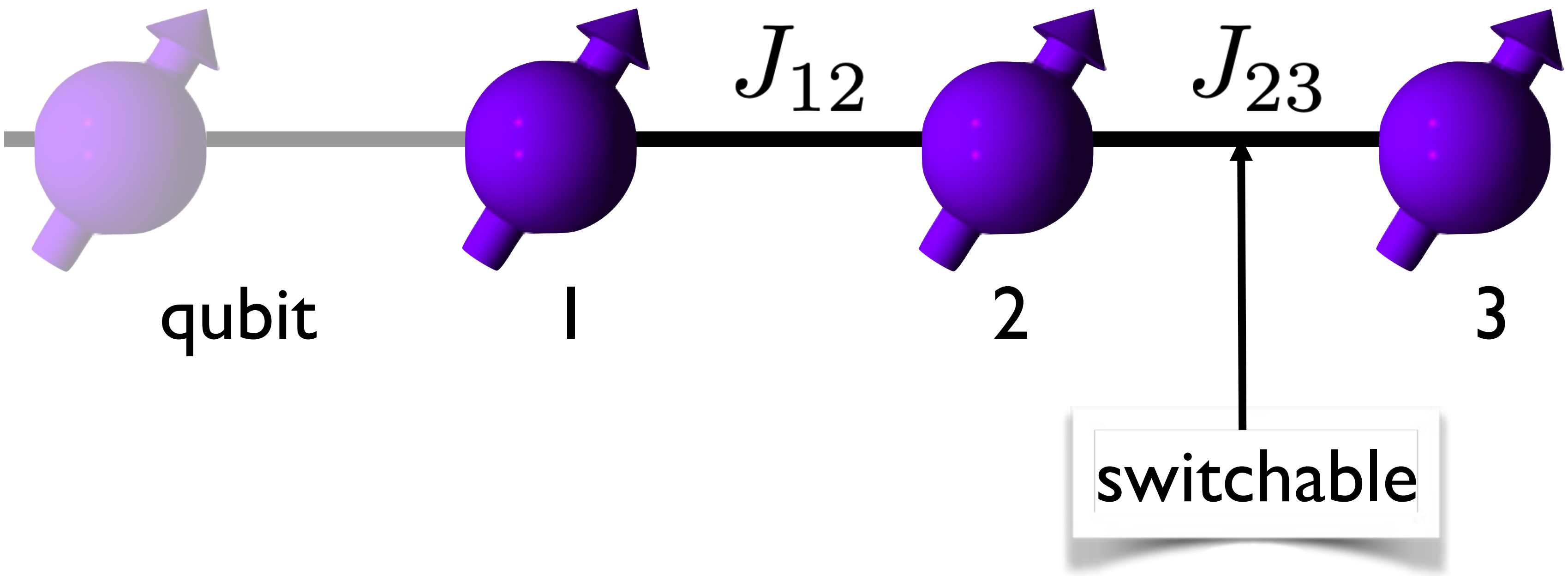
$$= 2\pi \sum_{i<j} J_{ij} \{ I_i^+ I_j^- + I_i^- I_j^+ \}/2,$$

State transfer in spin chains



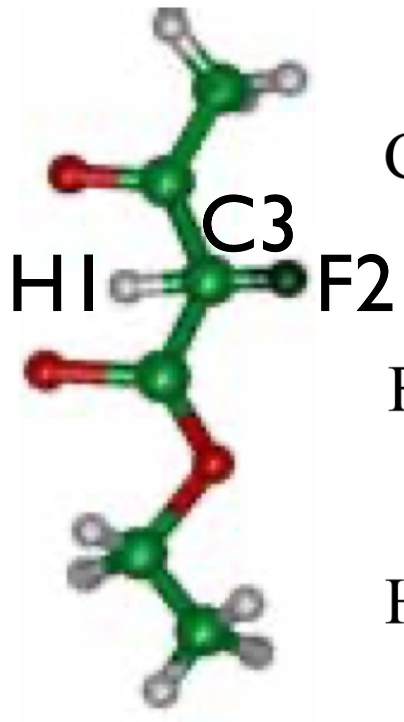
State Transfer

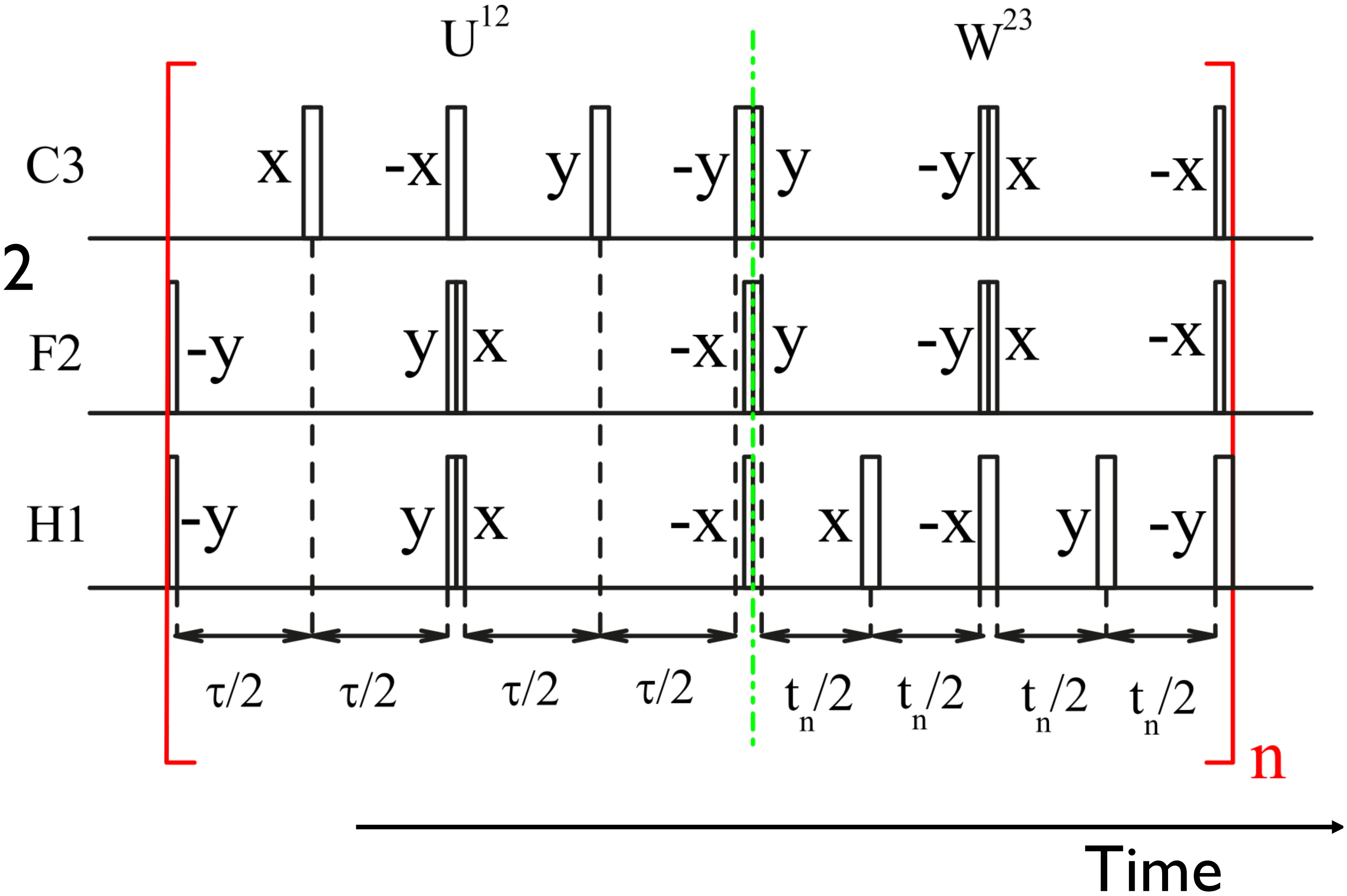
$$\mathcal{H} = 2\pi \sum_{i,k} J_{ik} \left(S_x^{(i)} S_x^{(k)} + S_y^{(i)} S_y^{(k)} \right)$$



How can quantum information be transferred along the chain?

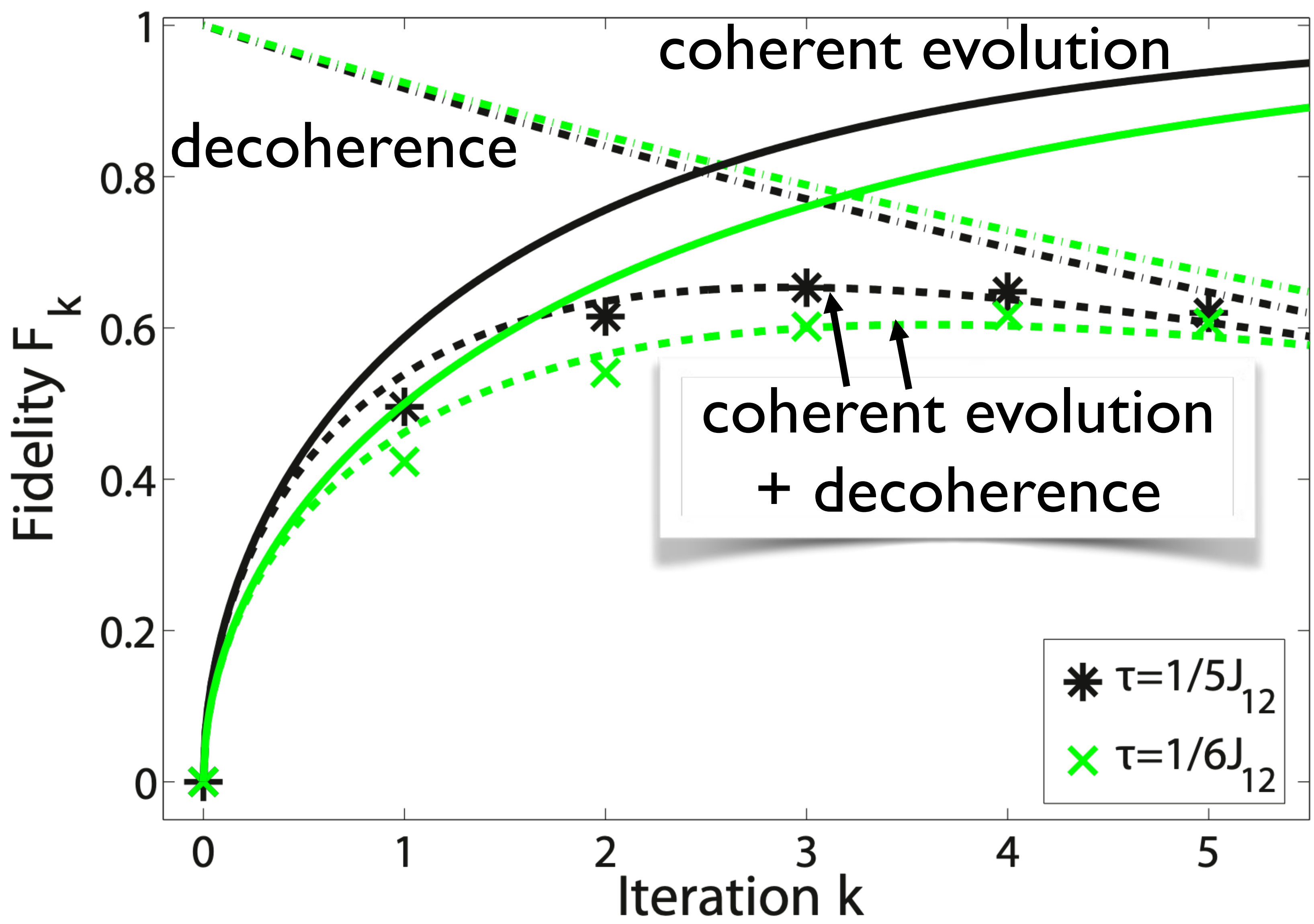
Pulse Sequence





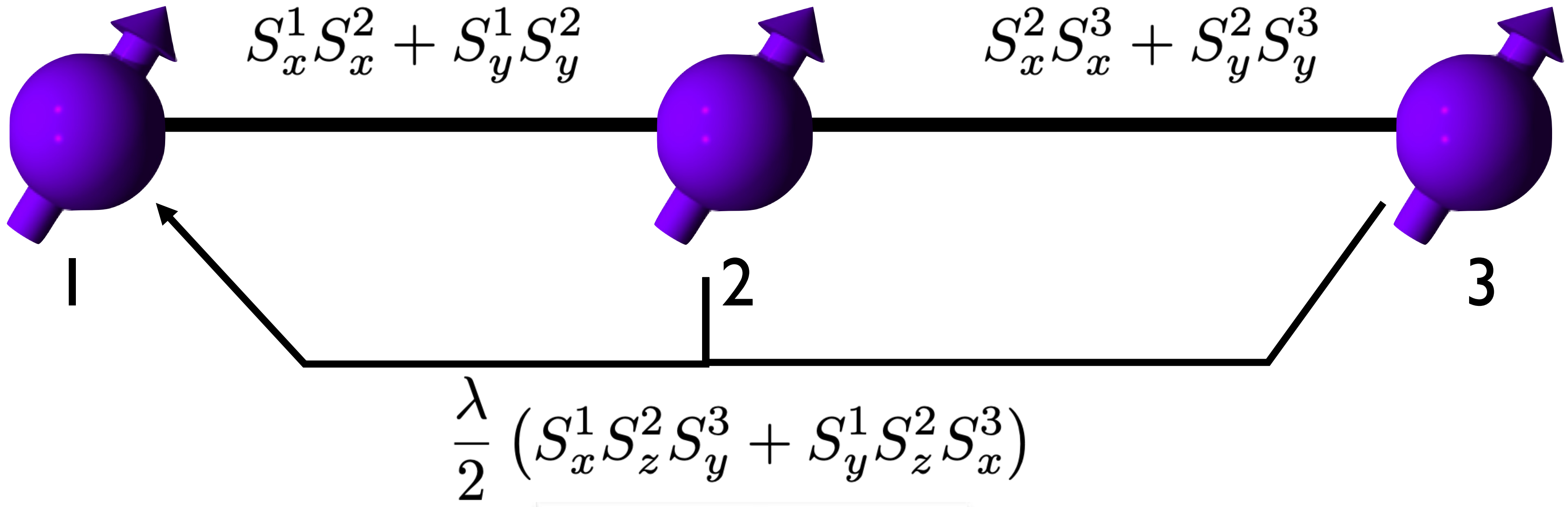
Phys. Rev. A 76, 012317 (2007).

Transfer Achieved

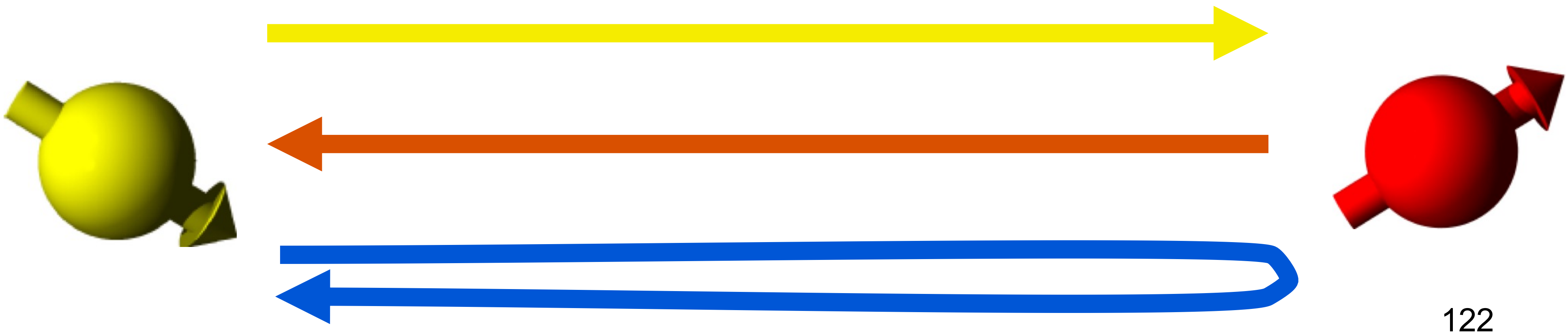


Phys. Rev. A *76*, *012317 (2007).*

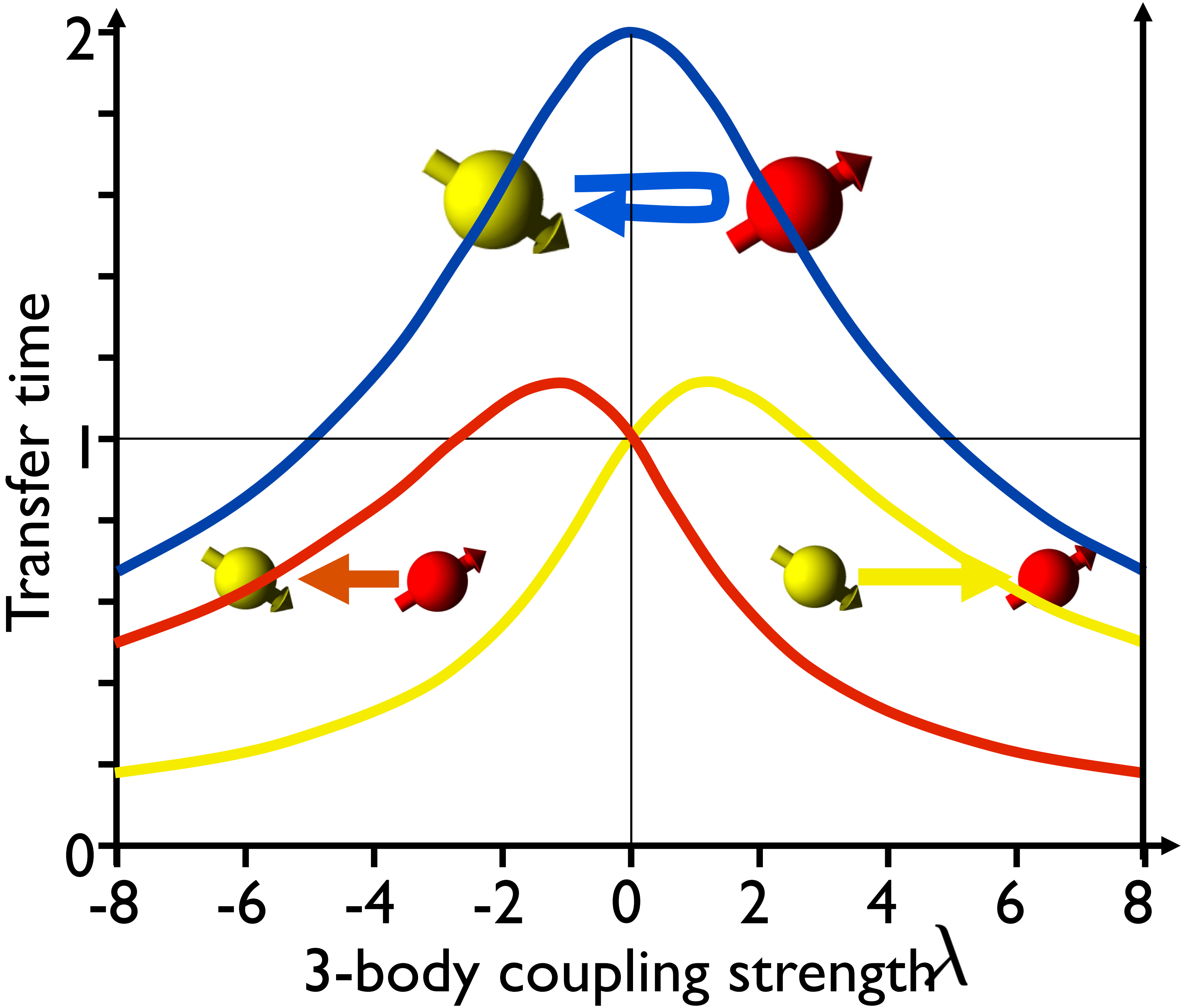
3-Body Interaction



State transfer:

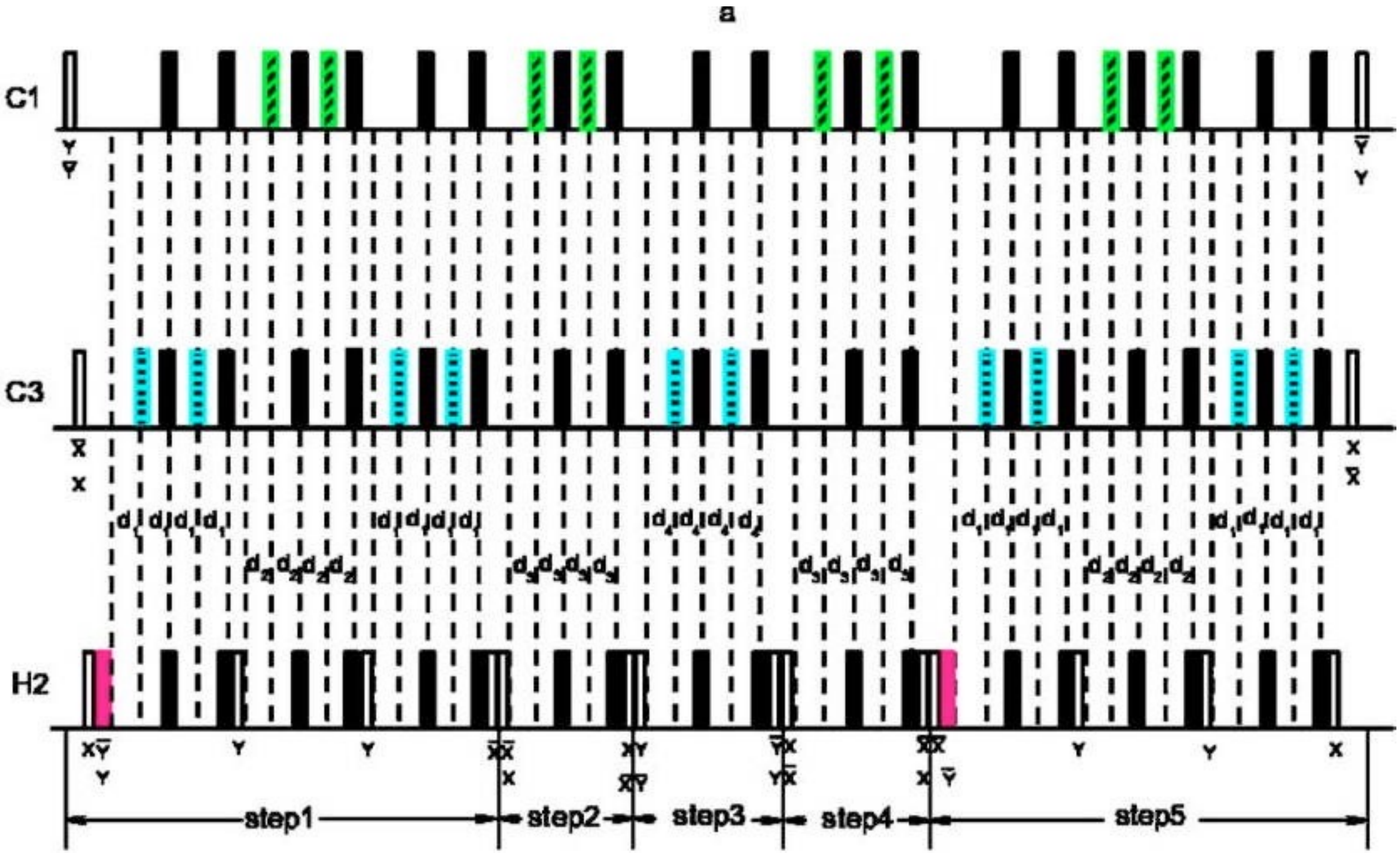


Transfer Speed

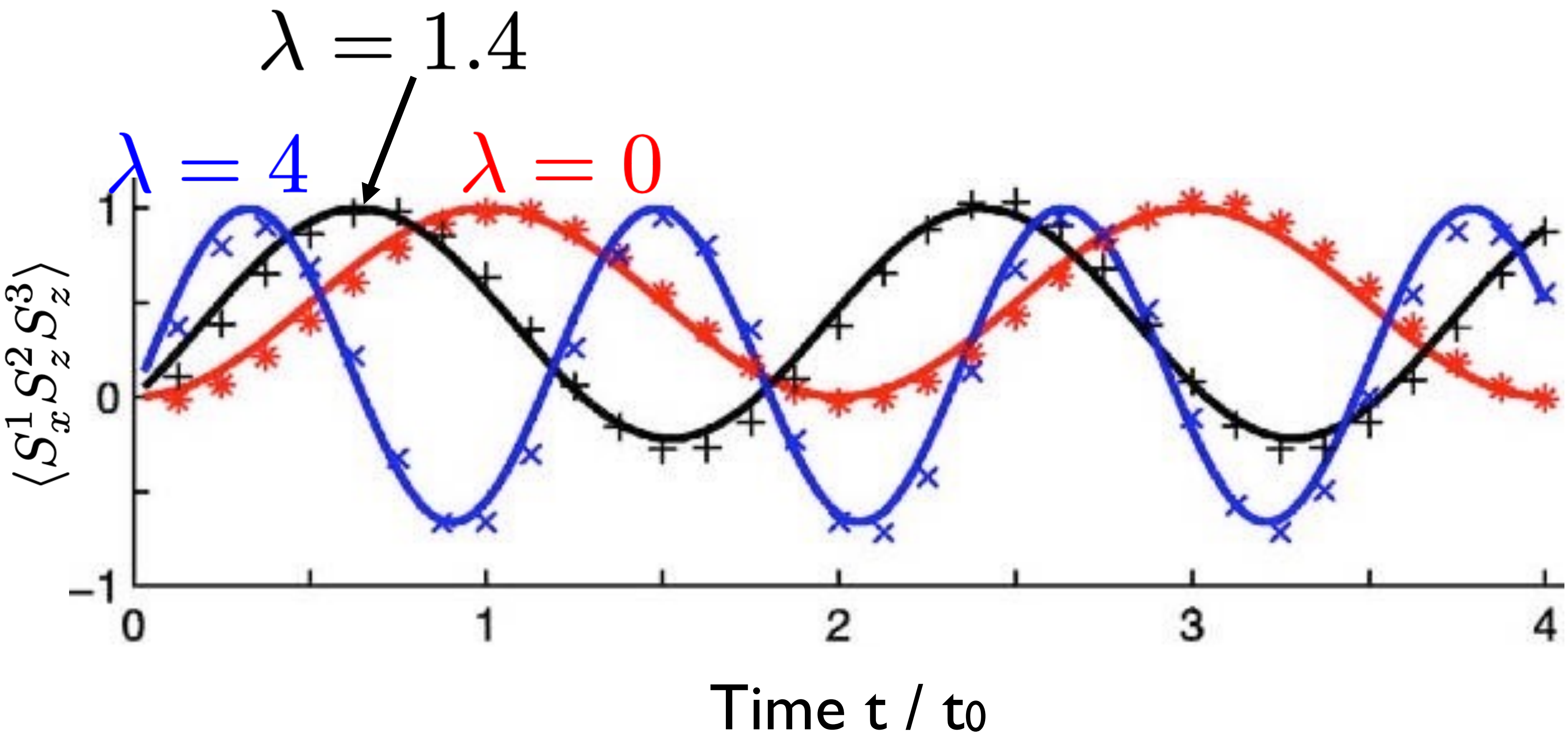


Implementation

"Half of Hamiltonian"



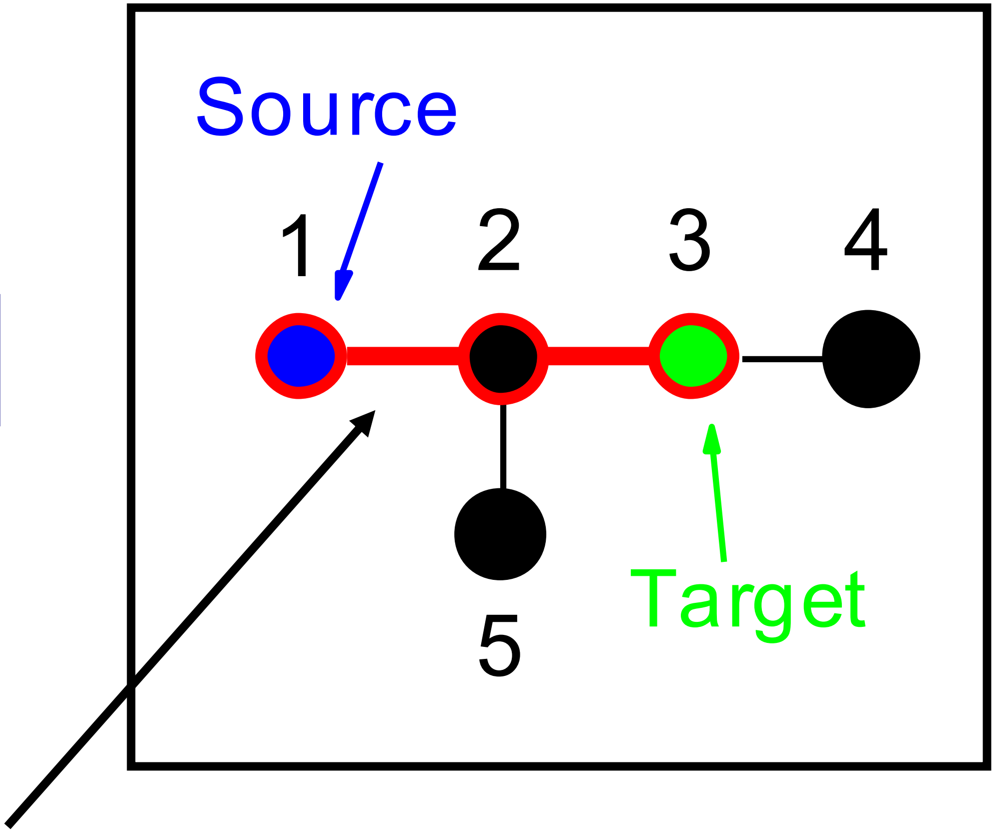
Evolution



Phys. Rev. A, 73, 062325 (2006).

Selective state transfer with global gates

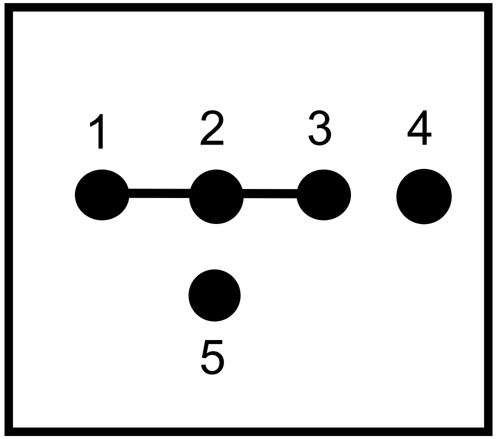
Nuclear spin ½ - qubits



Selective transfer

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



L-leucina

$$\hat{H} = \hat{H}_Z + \hat{H}_{\text{mix}}$$

The system Hamiltonian

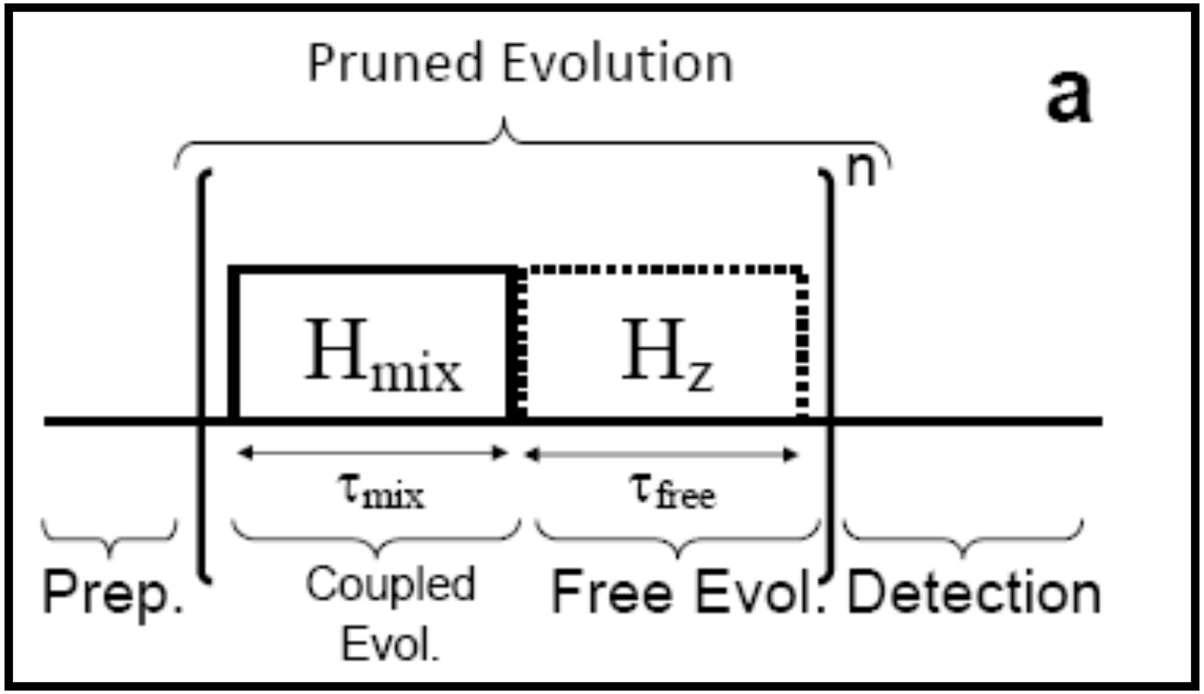
$$\hat{H}_Z = \sum_i \hbar\left(\Omega_0 + \Delta\Omega_i\right)\hat{I}_i^z$$

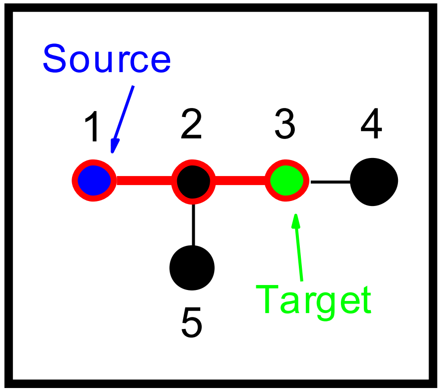
Zeeman Hamiltonian
Free evolution

$$\hat{H}_{\text{mix}} = \hat{H}_{XY} = \sum_{i,j} J_{ij}\left(\hat{I}_i^x \hat{I}_j^x + \hat{I}_i^y \hat{I}_j^y\right)$$

Mixing Hamiltonian – Interaction Hamiltonian

Selective state transfer with global gates





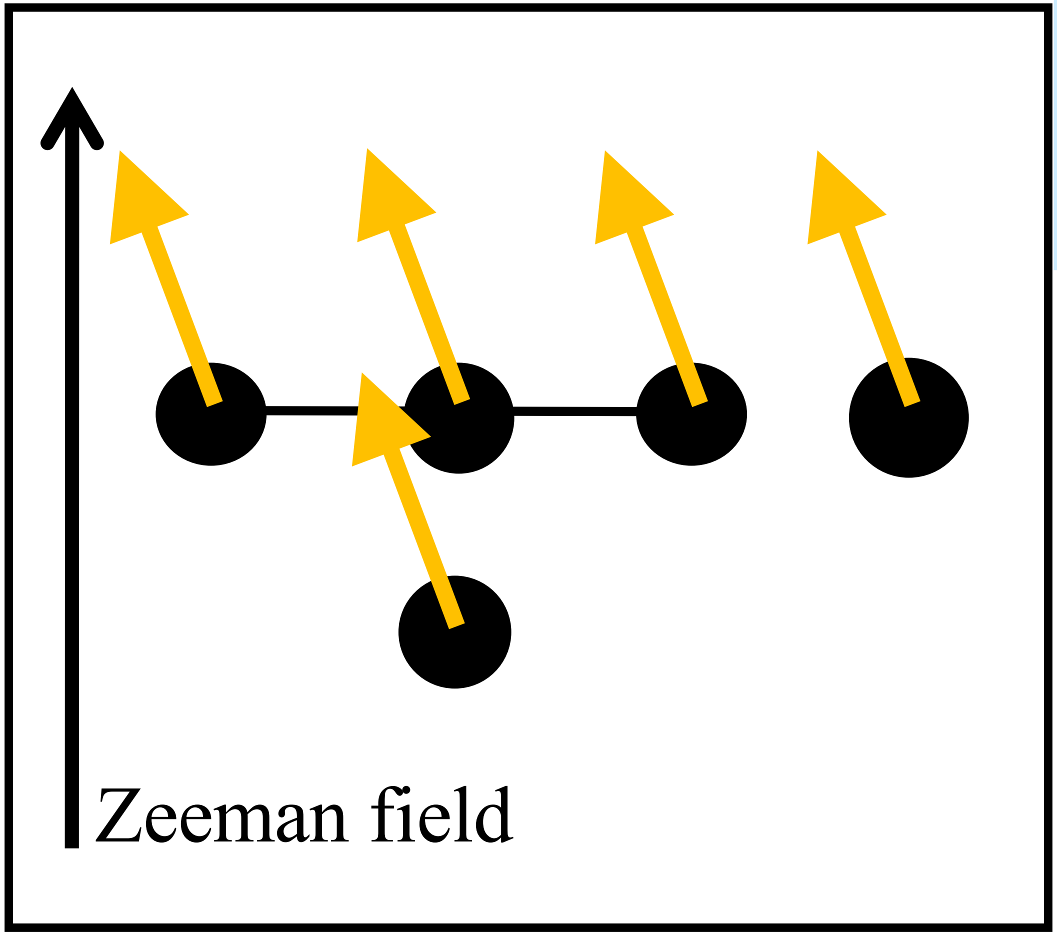
$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

i,j belong to the spins of the selected path

$$k_{ij} \in \mathrm{N}$$

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



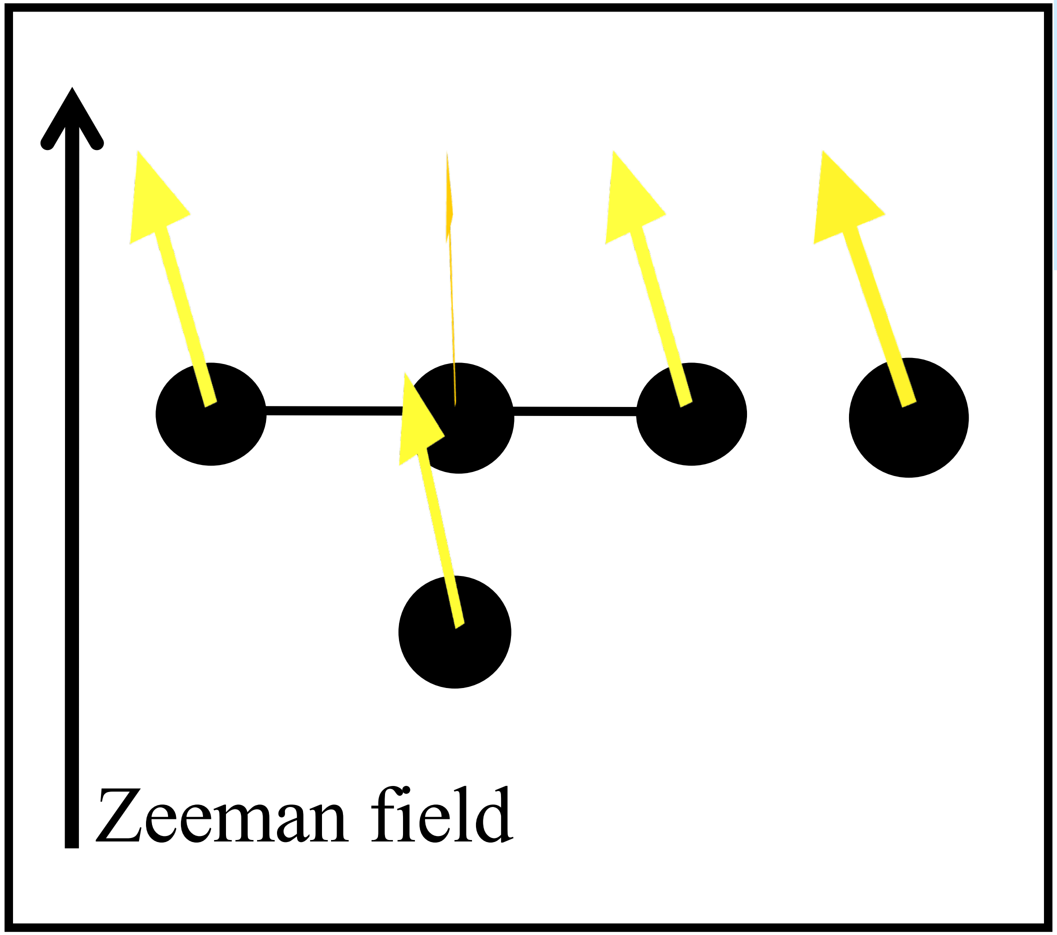
$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

i,j belong to the spins of the selected path

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



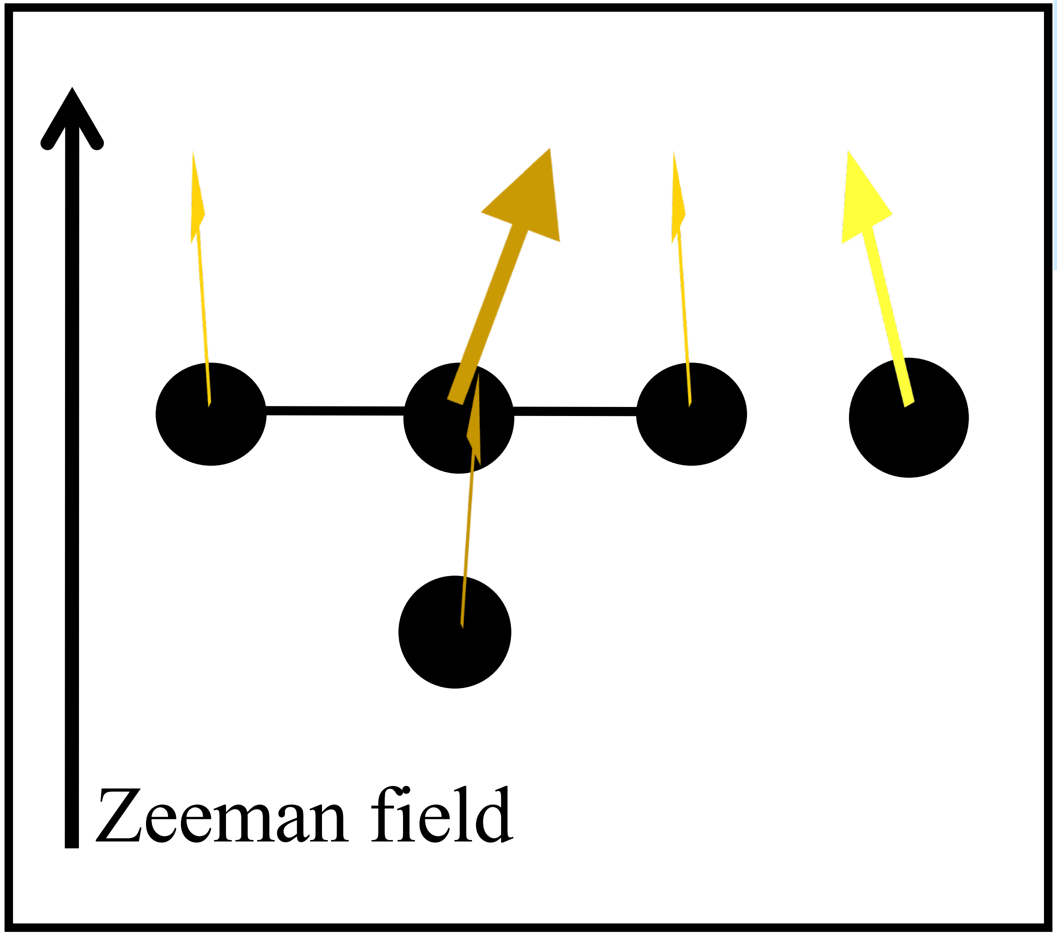
$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

i,j belong to the spins of the selected path

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



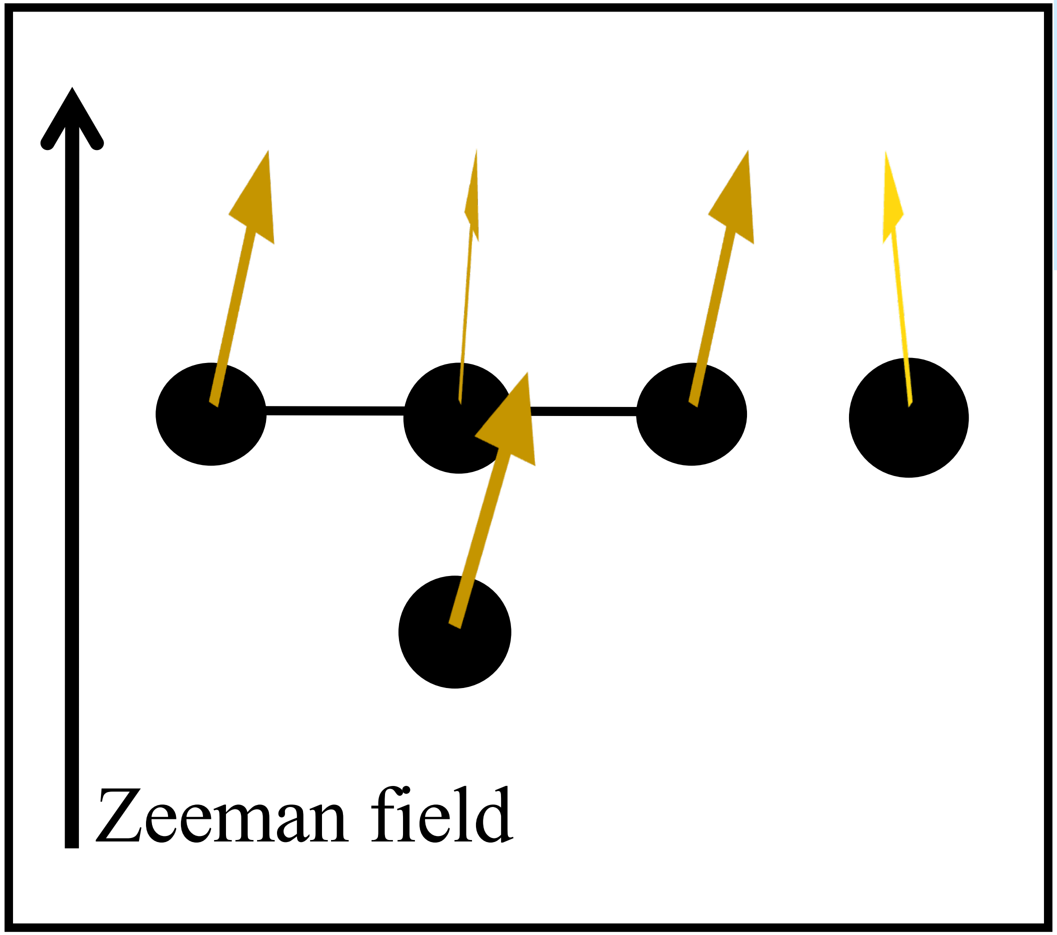
$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

i,j belong to the spins of the selected path

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



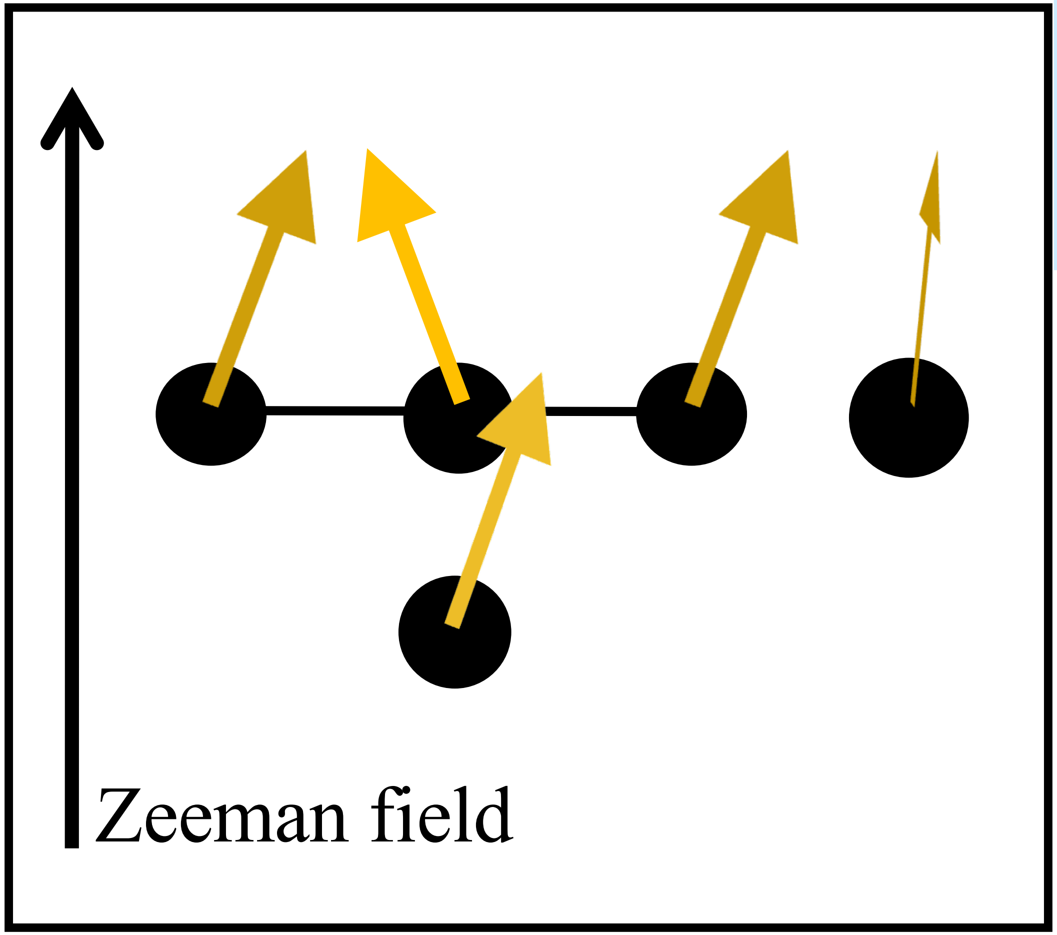
$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

i,j belong to the spins of the selected path

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



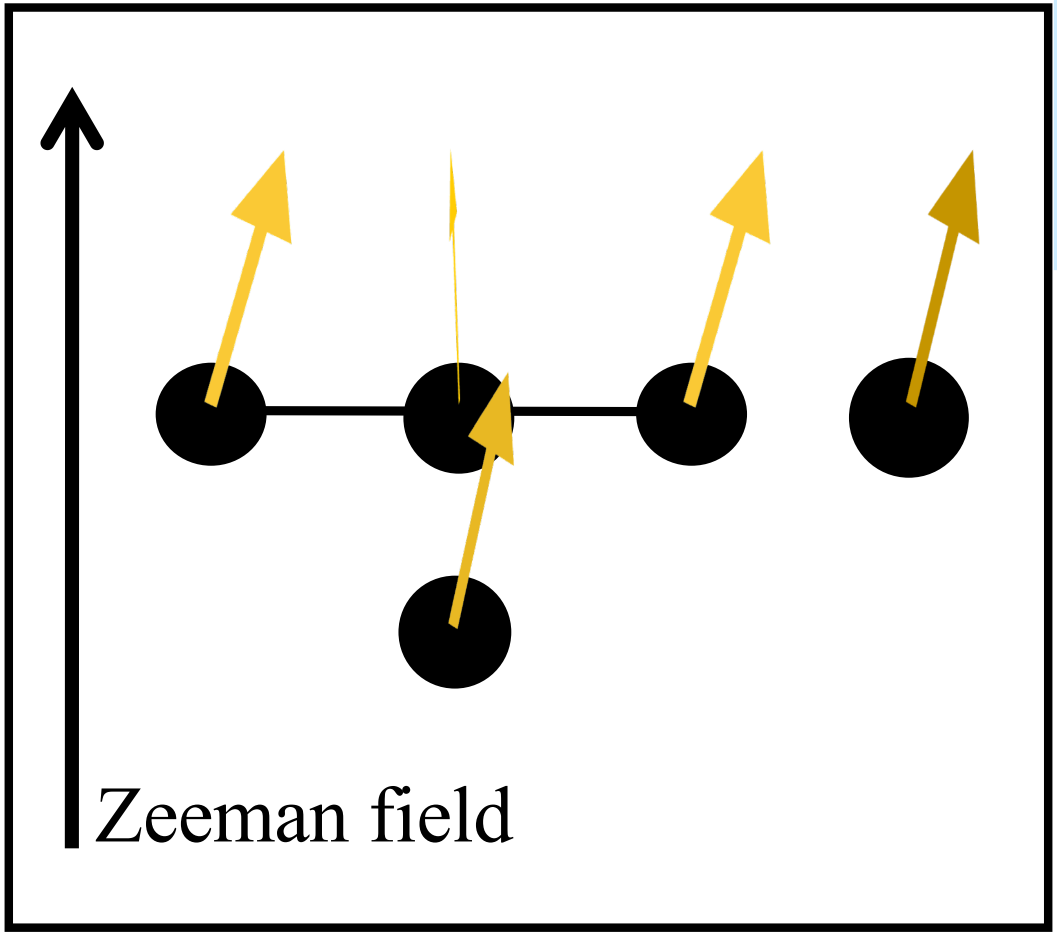
$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

i,j belong to the spins of the selected path

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



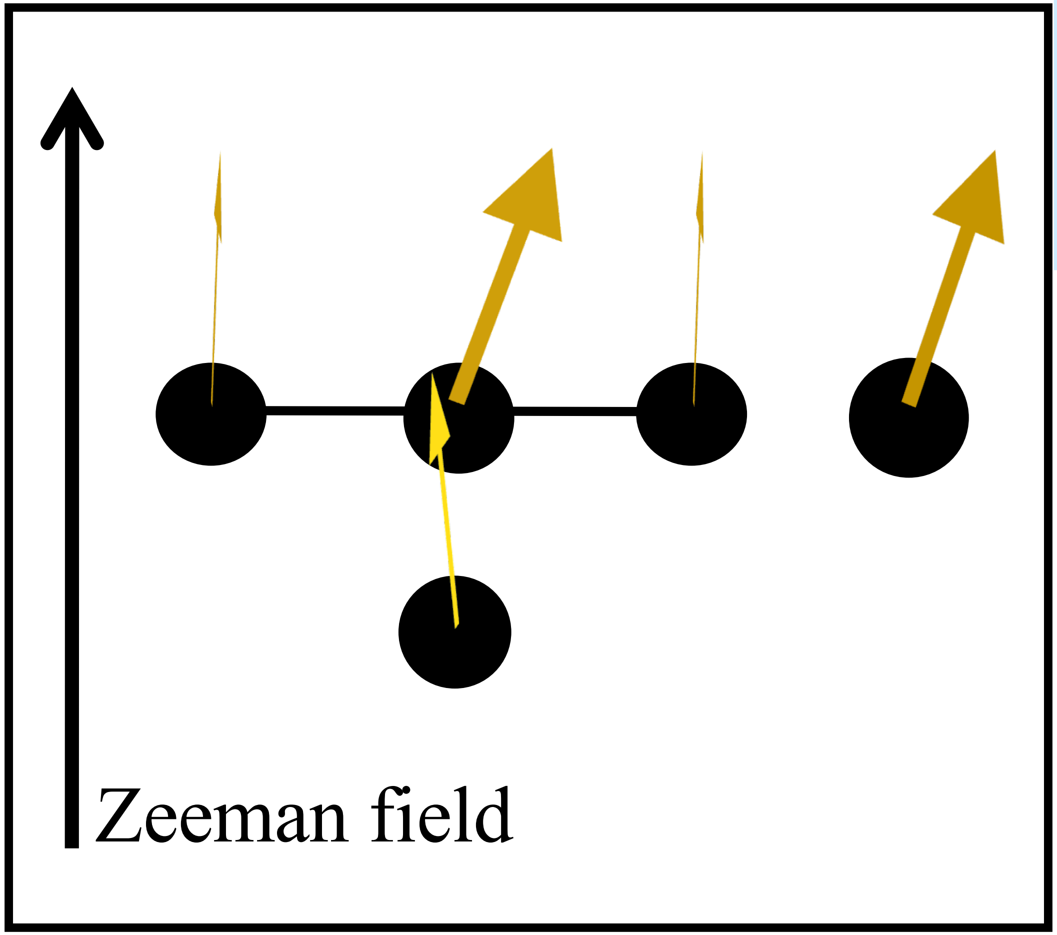
$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

i,j belong to the spins of the selected path

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



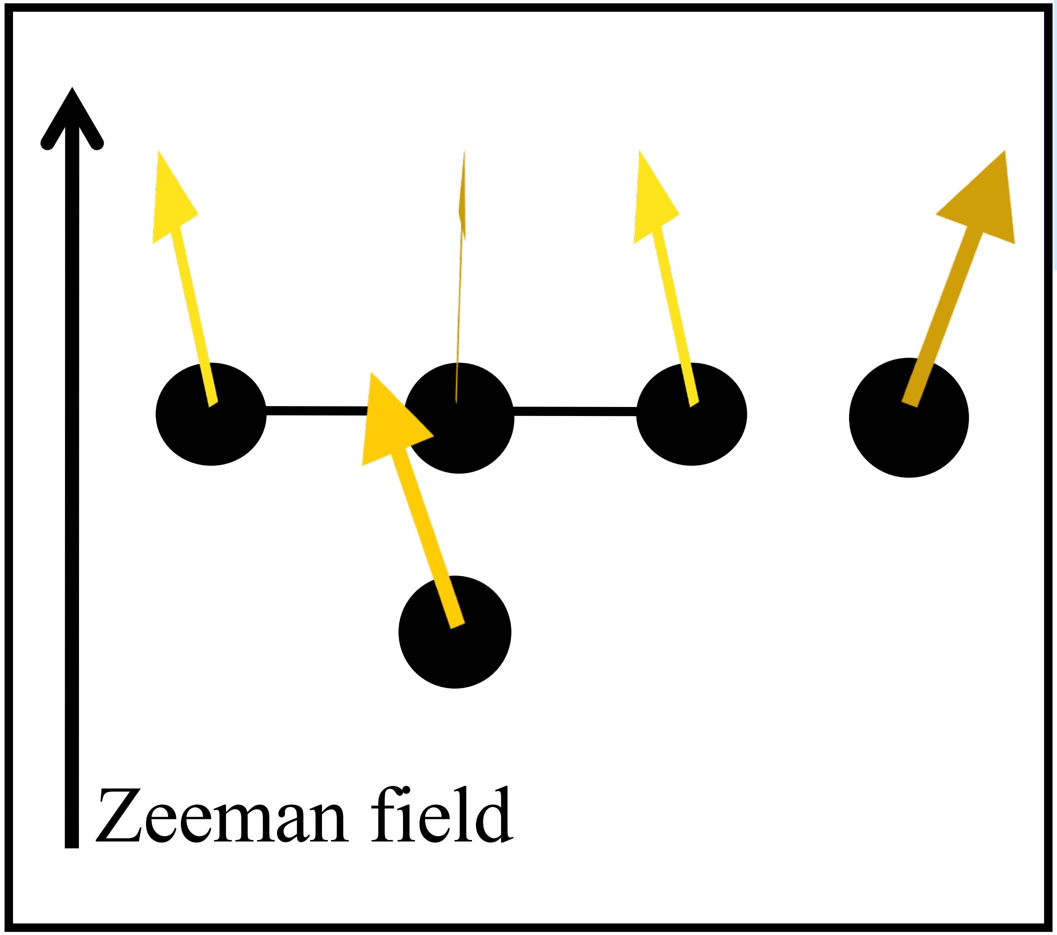
$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

i,j belong to the spins of the selected path

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



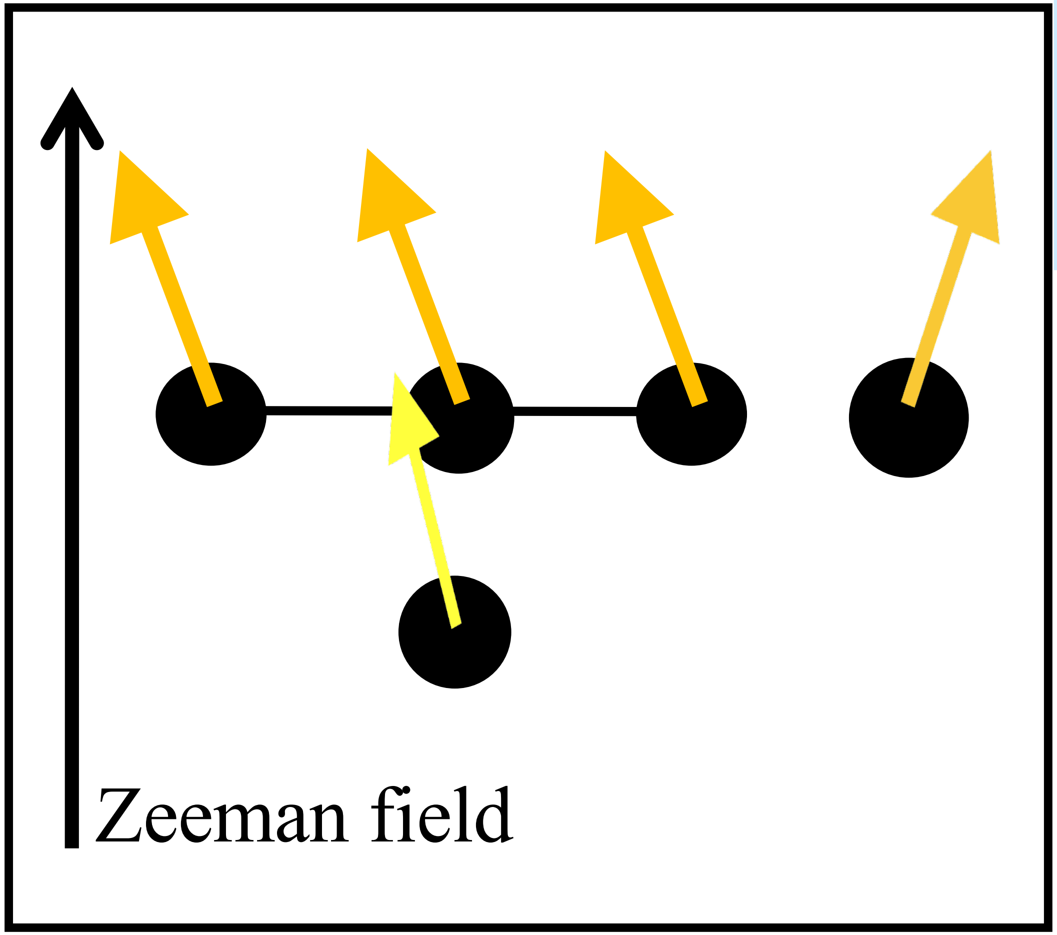
$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

i,j belong to the spins of the selected path

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



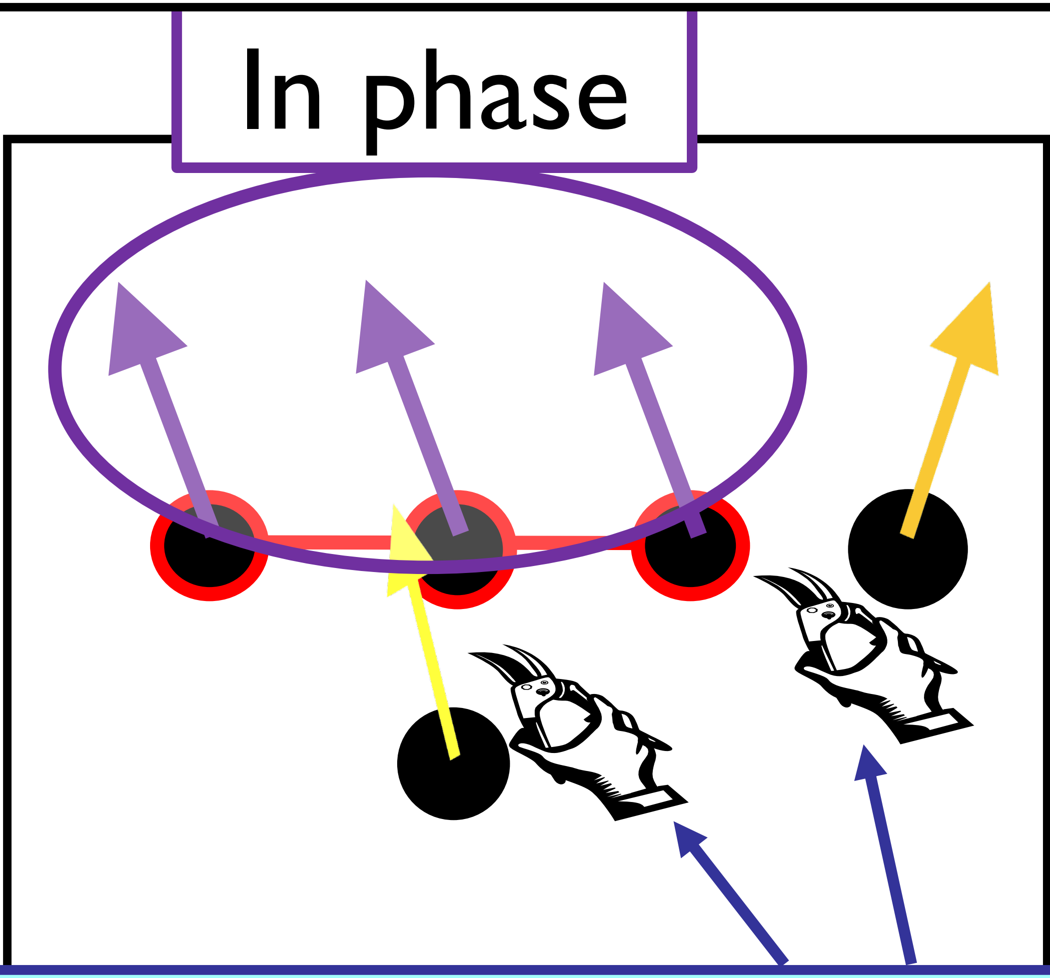
$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

i,j belong to the spins of the selected path

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer with global gates



$$\Delta t_{free} = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

i,j belong to the spins of the selected path

They are effectively pruned by successive interruptions

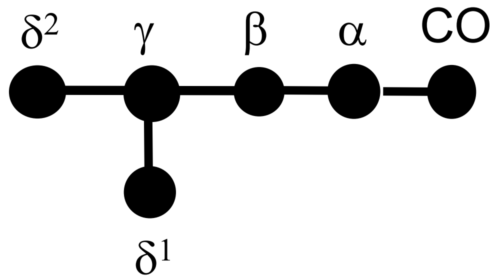
G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

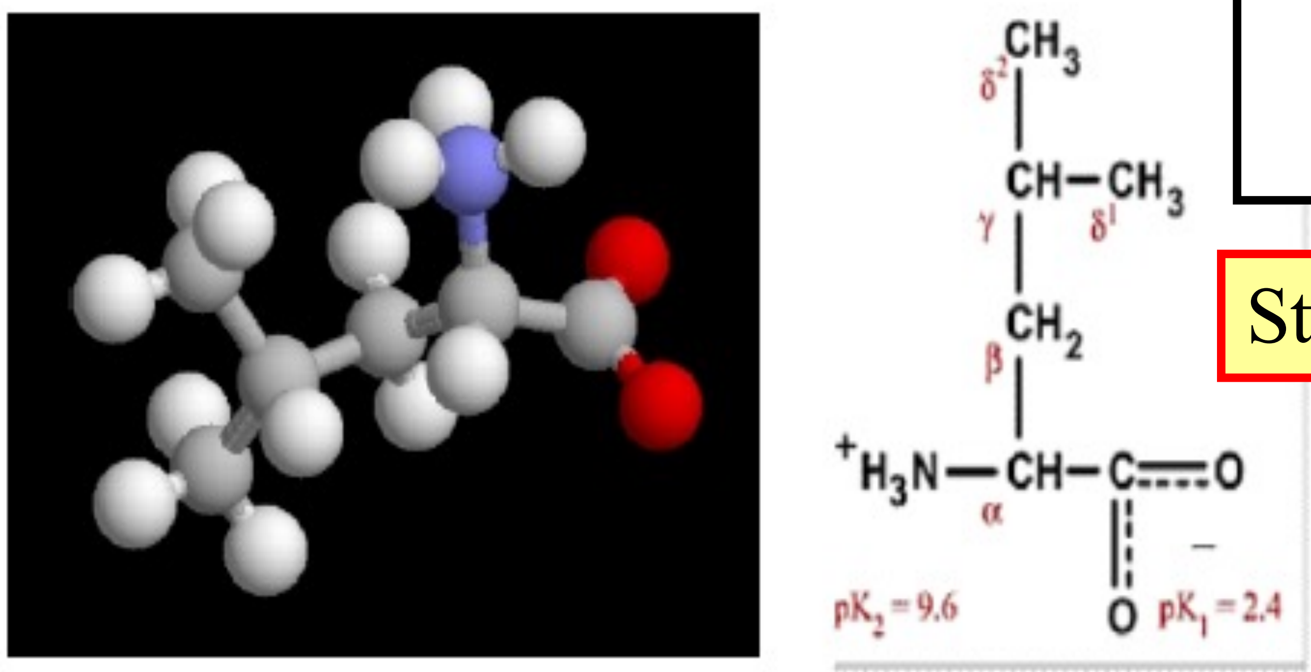
NMR quantum Simulator (liquid state)

XY effective Hamiltonian

$$\left|\Delta\Omega_{\mathrm{i}} - \Delta\Omega_{\mathrm{j}}\right| >> J_{ij}$$

2-3 spins → Perfect State Transfer



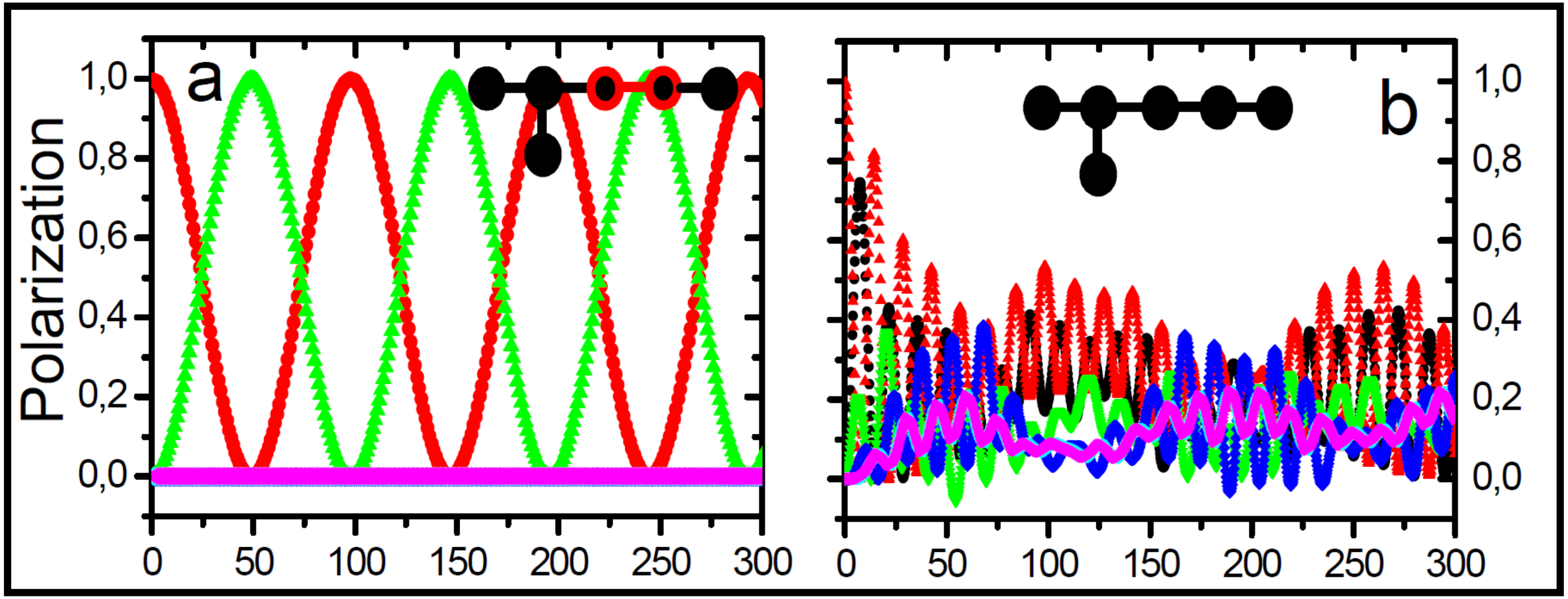


Step by step transfer

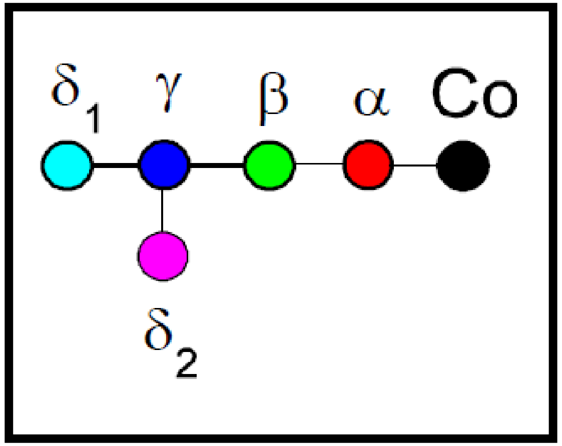
L-leucina

G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Directional Perfect State Transfer

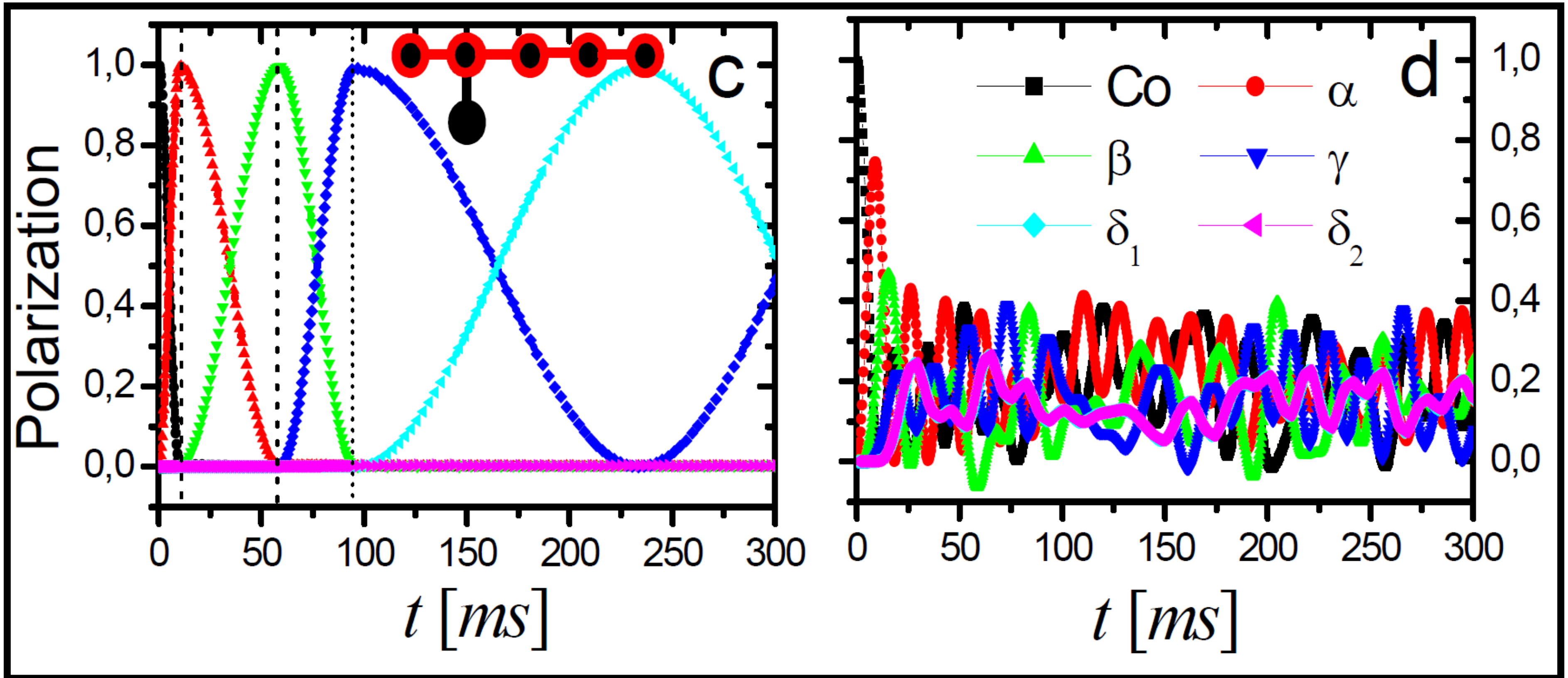


$\Delta\Omega_{Co}=2\pi 22.22\text{kHz}$; $\Delta\Omega_{\alpha}=2\pi 6.58\text{kHz}$; $\Delta\Omega_{\beta}=2\pi 5.21\text{kHz}$;
$\Delta\Omega_{\gamma}=2\pi 3.13\text{kHz}$; $\Delta\Omega_{\delta 1}=2\pi 2.74\text{kHz}$; $\Delta\Omega_{\delta 2}=2\pi 2.88\text{kHz}$
$J_{Co,\alpha}=2\pi 59\text{Hz}$; $J_{\alpha,\beta}=J_{\beta,\gamma}=J_{\gamma,\delta 1}=J_{\gamma,\delta 2}=2\pi 35\text{Hz}$

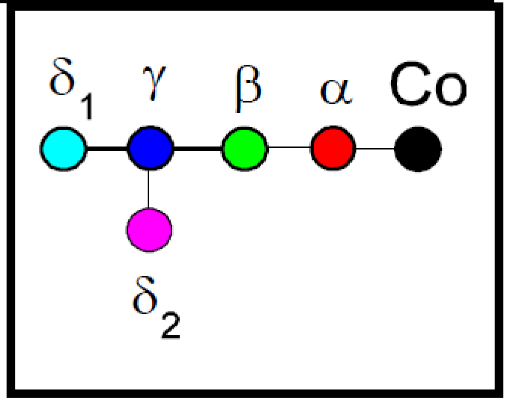


G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Directional Perfect State Transfer

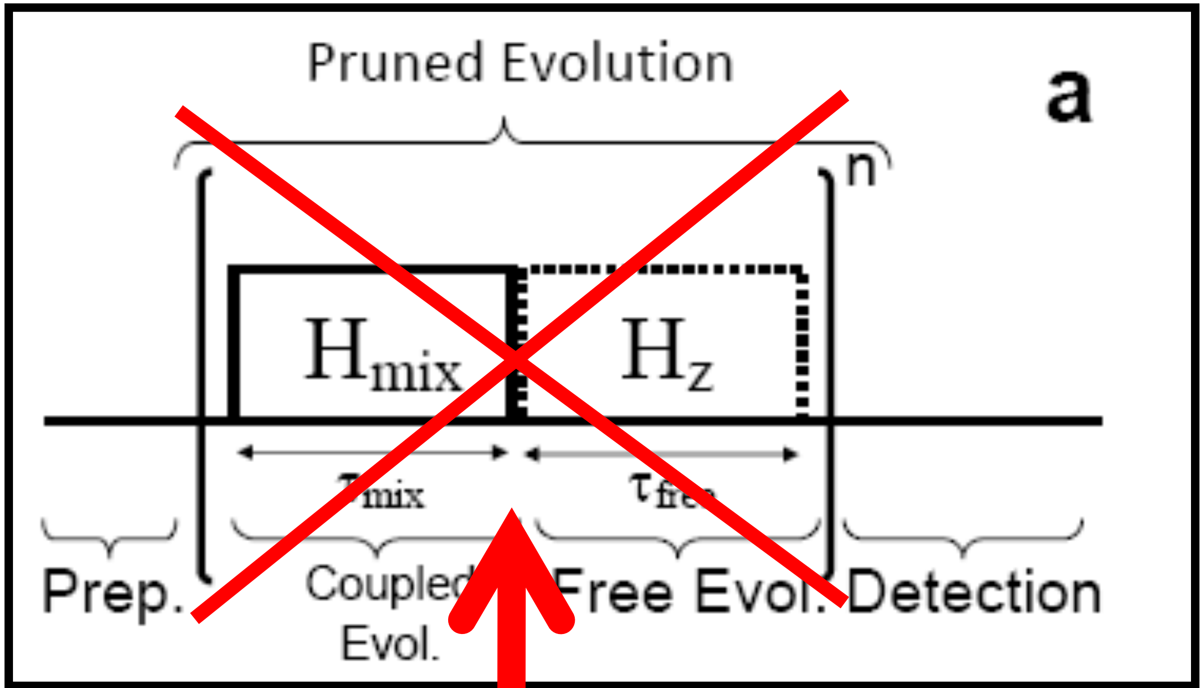


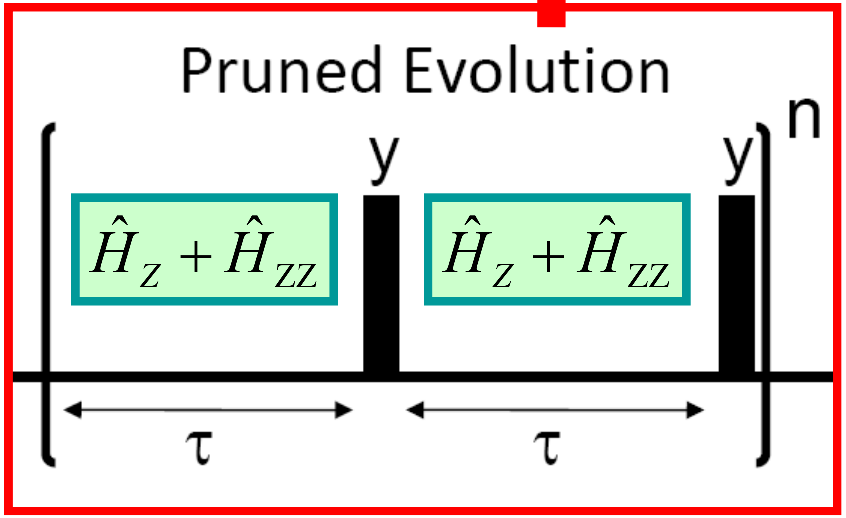
$\Delta\Omega_{Co}=2\pi 22.22\text{kHz}$; $\Delta\Omega_{\alpha}=2\pi 6.58\text{kHz}$; $\Delta\Omega_{\beta}=2\pi 5.21\text{kHz}$;
$\Delta\Omega_{\gamma}=2\pi 3.13\text{kHz}$; $\Delta\Omega_{\delta 1}=2\pi 2.74\text{kHz}$; $\Delta\Omega_{\delta 2}=2\pi 2.88\text{kHz}$
$J_{Co,\alpha}=2\pi 59\text{Hz}$; $J_{\alpha,\beta}=J_{\beta,\gamma}=J_{\gamma,\delta 1}=J_{\gamma,\delta 2}=2\pi 35\text{Hz}$

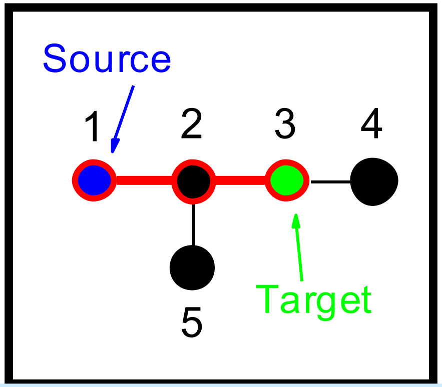


G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Selective state transfer







$$\tau = \frac{2\pi k_{ij}}{\left|\Delta\Omega_i - \Delta\Omega_j\right|}$$

$$k_{ij} \in \mathrm{N}$$

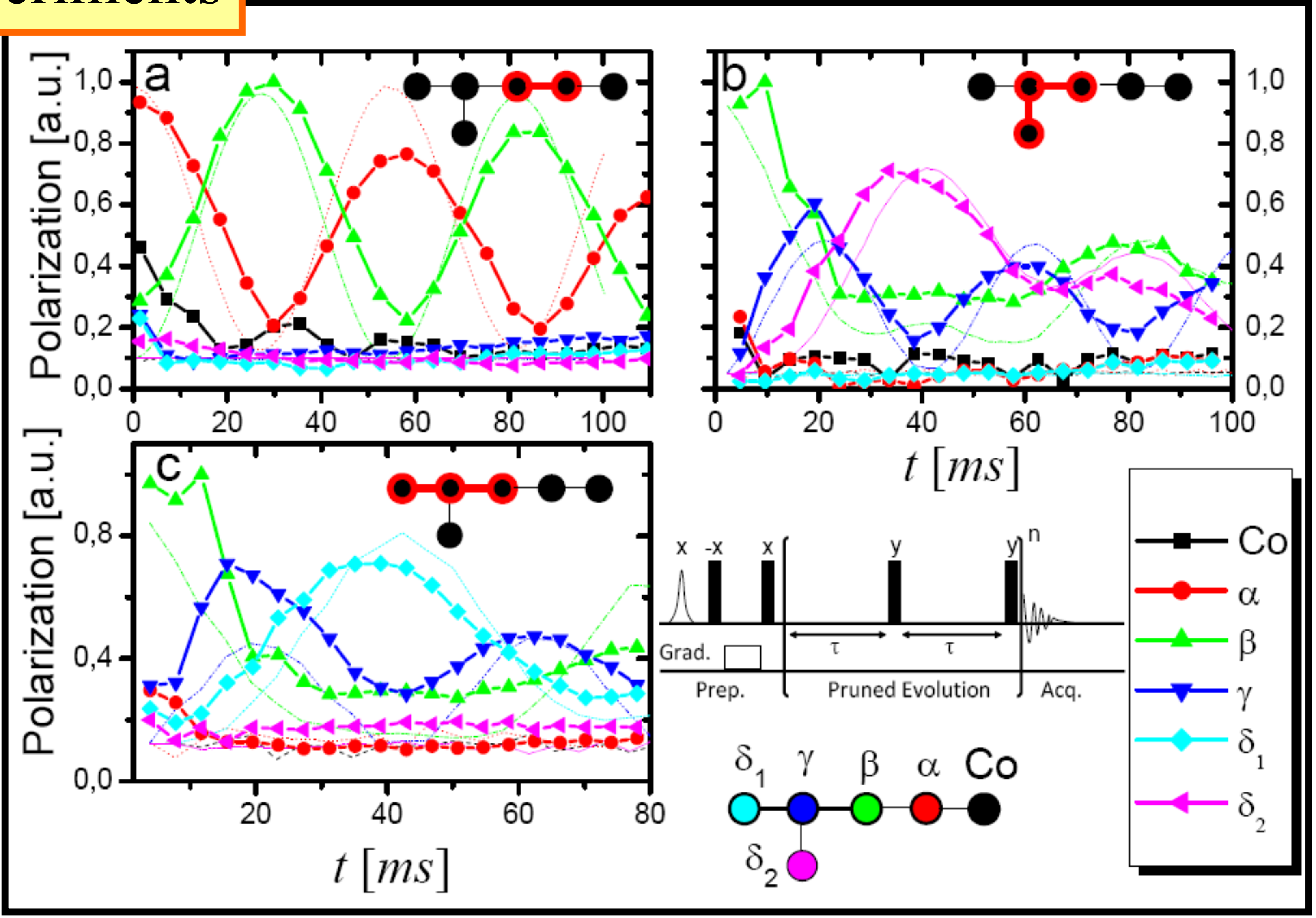
$$\hat{H} = \hat{H}_Z + \hat{H}_{\mathrm{mix}}$$

$$\hat{H}_{\mathrm{mix}} = \hat{H}_{ZZ} = \sum_{i,j} J_{ij} \hat{I}_i^z \hat{I}_j^z$$

$$\left|\Delta\Omega_{\mathrm{i}} - \Delta\Omega_{\mathrm{j}}\right| >> J_{ij}$$

Directional Perfect State Transfer

Experiments



G.A. Álvarez, et al. Phys. Rev. A (R) **81**, 060302 (2010)

Exercise

Show that the **SWAP operation** between two interacting spins governed by the Hamiltonian $\mathcal{H} = -\, \omega_A\, A_z - \omega_X\, X_z + d\, A_z\, X_z$ can be implemented in **liquid-state NMR** using **selective π-pulses**. Consider the ideal limit for the selective pulses, i.e. the selective **π** rotations are ideal.

Use the following decomposition, discussed in class:

SWAP=CNOT(1,2)·CNOT(2,1)·CNOT(1,2)

Assume that each **CNOT(a, b)** denotes a CNOT gate with **spin *a* as the control** and **spin *b* as the target**, and explain how each of these gates can be implemented using **NMR pulse sequences**, based on **selective π-pulses**.