

Joachim Stolze, Dieter Suter

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Quantum Computing

A Short Course from Theory to Experiment

Second, Updated and Enlarged Edition

Stolze · Suter

Quantum Computing 2nd Edition

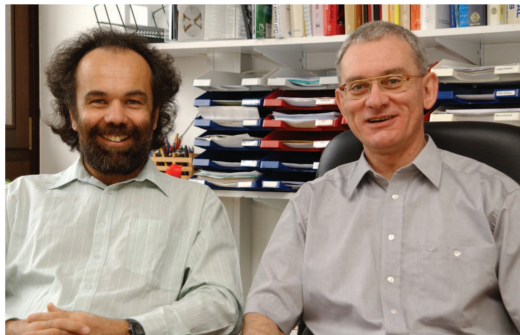
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The book gives an impressive description of current experiments. ... Despite of the high density of information, the authors succeeded to describe the fundamental concepts and methods. As it contains numerous references and links, this book also forms a solid starting point for further studies.”

Physik Journal on the first edition

New to this edition

- Working with single photons
- Report on progress in the trapping and manipulation of neutral particles
- Quantum simulations of strongly correlated many-particle models
- End-of-chapter problems



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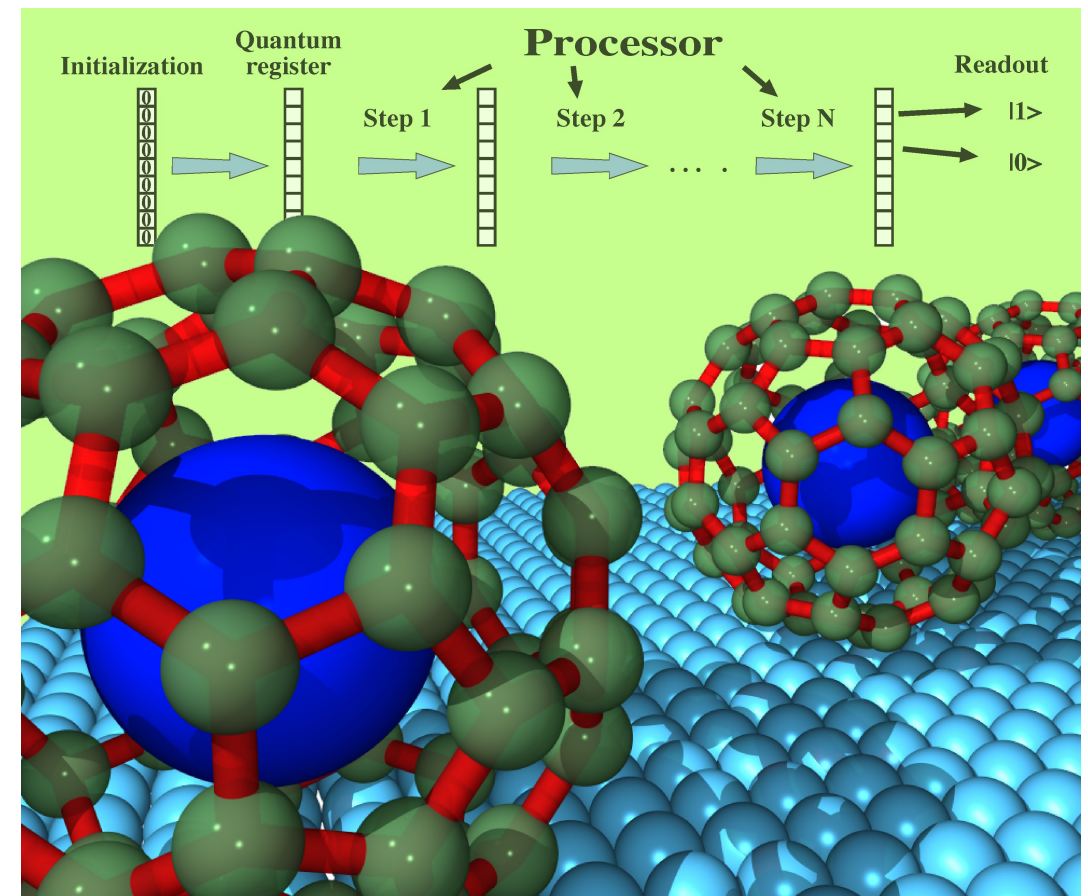
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ISBN 978-3-527-40787-3



www.wiley-vch.de

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WILEY-VCH Verlag Berlin GmbH
October 25, 2007

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