MOLECULAR PHYSICS, 1993, VOL. 78, No. 1, 95-102

Quantum time-translation machine
An experimental realization

By DIETER SUTER[1], MATTHIAS ERNST[2] and RICHARD R. ERNST[2]

[1]Institut für Quantenelektronik and [2]Laboratorium für Physikalische Chemie, Swiss Federal Institute of Technology (ETH) Zurich, CH-8093 Zurich, Switzerland

(Received 12 May 1992; accepted 15 June 1992)

We report for the first time an experimental realization of a 'quantum time-translation machine' via superpositions of time evolutions as suggested by Aharonov, Anandan, Popescu and Vaidman (1990, *Phys. Rev. Lett.*, 64, 2965). The physical system consists of two coupled spins 1/2 and the amplification scheme is applied to the heteronuclear coupling between the two spins. We give a simple explanation of the phenomenon and suggest possible classical analogues.

In a recent communication [1], Aharonov and coworkers introduced a new notion: 'a superposition of time evolutions (rather than of states) of a quantum system'. These superpositions are created by coupling a quantum mechanical system S to an external quantum mechanical system I in such a way that the time evolution of system S depends on the state of system I. During a short time, the two coupled systems are allowed to evolve freely. After this evolution period, the state of the system S depends on a quantum variable of system I represented by the eigenvalues of an operator I_α acting only on system I. The superposition of time evolutions of system S is then obtained by forcing system I into a state that is a superposition of the eigenstates of I_α. This projection of the state of system 1 affects the subsystem S via the entanglement that was established during the free evolution of the coupled systems. The resulting state of the system S can then differ considerably from any state that it could have reached by free evolution during the same time, but may be similar to a state that would have resulted from a prolonged evolution under the same Hamiltonian. Aharonov and coworkers therefore compared the effect to that of a time-translation machine.

In this paper we give a demonstration that such a time translation can actually be realized in the laboratory. Our experimental system resembles closely one of the examples suggested by Aharonov *et al.*, consisting of an ensemble of spins $S=1/2$ in a magnetic field whose strength depends on a quantum mechanical space coordinate. In our experiment, the strength of the magnetic field acting on the system S is not correlated to a spatial coordinate, but to the state of a second spin I. More specifically, our system consists of two spins $S=1/2$ and $I=1/2$ that are coupled to each other via an Ising-type interaction:

$$\mathcal{H}_{SI} = 2\pi J S_z I_z, \qquad (1)$$

where S_z and I_z are the z components of the spin angular momentum operators acting on the two spins. The observed spin S evolves under the influence of the heteronuclear coupling to the spin I which can be in the eigenstate $|\alpha\rangle$ or $|\beta\rangle$ of the operator I_z. The superposition corresponding to equation (3) of reference [1] is then obtained by changing the quantization axis of spin I from z to z'. The observed physical quantity is the orientation of the transverse magnetization of spin S when spin I is in either one of the two eigenstates of $I_{z'}$.

If spin S is initially aligned along the x axis, its wavefunction can be expanded in terms of the eigenfunctions of S_z as $\Psi(0)=2^{-1/2}$ (1, 1). In a magnetic field parallel to the quantization axis, the spin precesses around the magnetic field; in terms of the state Ψ, this corresponds to an evolution $\Psi(t)=2^{-1/2}(\mathrm{e}^{-i\delta/2}, \mathrm{e}^{i\delta/2})$ where the phase δ is given by $\delta=-\gamma_S B_0 t$, γ_S represents the gyromagnetic ratio of spin S and B_0 the strength of the magnetic field. If, as in our case, the magnetic field strength depends on the external quantum coordinate I_z, the sense of precession of the spin is correlated to this coordinate and the phase acquired by the spin depends now on the oeprator I_z as $\delta_I=2\pi J t I_z$, due to the 'magnetic field' $B_I=-(2\pi J/\gamma_S)I_z$. In order to make this dependence on the external system more explicit, we turn now to a description of this system in terms of the total state space of the combined SI spin system. Using the basis $|\alpha\alpha\rangle, |\alpha\beta\rangle, |\beta\alpha\rangle, |\beta\beta\rangle$, we choose the initial wavefunction as

$$\Psi_{SI}(0) = 1/2(1, 1, 1, 1). \tag{2}$$

Evolution under the Hamiltonian (1) yields then $\Psi_{SI}(t)=\frac{1}{2}(\mathrm{e}^{-i\delta/2}, \mathrm{e}^{i\delta/2}, \mathrm{e}^{i\delta/2}, \mathrm{e}^{-i\delta/2})$ where the phase δ is now $\delta=\pi J t$. The superposition of time evolutions can now be obtained by projecting this state onto the eigenfunctions of an operator acting on the I subsystem that does not commute with I_z. For the spin 1/2 system considered here, the most general such operator can be written as $I_{\theta,\phi}=\boldsymbol{I}\cdot\boldsymbol{e}$ where $\boldsymbol{I}$ represents the spin angular momentum vector operator and e a unit vector in ^{3}. The eigenfunctions of this operator are $\zeta_{1/2}^{\theta,\phi}=$ (cos $(\theta/2)\,\mathrm{e}^{-i\phi/2}$, sin $(\theta/2)\,\mathrm{e}^{i\phi/2}$) and $\zeta_{-1/2}^{\theta,\phi}=(-$ sin $(\theta/2)\,\mathrm{e}^{-i\phi/2}$, cos $(\theta/2)\,\mathrm{e}^{i\phi/2})$, where θ and ϕ are the spherical coordinates of the unit vector. If the I part of the system is rewritten in terms of these eigenfunctions while the s-spin functions are retained, the overall wavefunction becomes therefore

$$\Psi_{SI}^{\theta,\phi}(t)=\frac{1}{2}\begin{pmatrix}\alpha\zeta_{-1/2}^{\theta,\phi}\mathrm{e}^{-i\delta/2}\\ \alpha\zeta_{+1/2}^{\theta,\phi}\mathrm{e}^{i\delta/2}\\ \beta\zeta_{-1/2}^{\theta,\phi}\mathrm{e}^{i\delta/2}\\ \beta\zeta_{+1/2}^{\theta,\phi}\mathrm{e}^{-i\delta/2}\end{pmatrix}=\frac{1}{2}\cos\frac{\theta}{2}\begin{pmatrix}\mathrm{e}^{-i(\delta-\phi)/2}\\ \mathrm{e}^{i(\delta-\phi)/2}\\ \mathrm{e}^{i(\delta+\phi)/2}\\ \mathrm{e}^{-i(\delta+\phi)/2}\end{pmatrix}+\frac{1}{2}\sin\frac{\theta}{2}\begin{pmatrix}\mathrm{e}^{-i(\delta-\phi)/2}\\ -\mathrm{e}^{i(\delta-\phi)/2}\\ \mathrm{e}^{-i(\delta+\phi)/2}\\ -\mathrm{e}^{i(\delta+\phi)/2}\end{pmatrix}. \tag{3}$$

The time-translation effect is now obtained by projecting the system onto the part corresponding to one of the two states $\zeta_{\pm 1/2}^{\theta,\phi}$ of system Z; this corresponds to the postselection process of reference [l]. The corresponding wavefunctions $\Psi_{S,\pm 1/2}(t)$ are given by the sum of the first and third or the second and fourth components of $\Psi_{SI}^{\theta,\phi}(t)$, respectively:

$$\Psi_{S,\pm 1/2}(t)=\frac{1}{2}\begin{pmatrix}(\alpha\mathrm{e}^{-i\delta/2}+\beta\mathrm{e}^{i\delta/2})\zeta_{-1/2}^{\theta,\phi}\\ (\alpha\mathrm{e}^{i\delta/2}+\beta\mathrm{e}^{-i\delta/2})\zeta_{+1/2}^{\theta,\phi}\end{pmatrix} \tag{4}$$

The state of this system clearly depends on the, direction of the quantization axis e of system I. In our experiment, the observable quantities are the S-spin angular momentum components perpendicular to the z axis whose expectation values are

$$\langle S_x\rangle_{\pm 1/2}(t) = \langle \Psi_{S,\pm 1/2}(t)|S_x|\Psi_{S,\pm 1/2}(t)\rangle = \tfrac{1}{4}[\cos\delta \pm \cos\phi\sin\theta]$$
$$\langle S_y\rangle_{S,\pm 1/2}(t) = \langle \Psi_{S,\pm 1/2}(t)|S_y|\Psi_{S,\pm 1/2}(t)\rangle = \pm\tfrac{1}{4}\sin\delta\cos\theta. \quad (5)$$

In close analogy to the case of free precession, we can now define a phase $\delta'_{\pm 1/2}(t)$ of the transverse magnetization as

$$\delta'_{\pm 1/2}(t) = \tan^{-1}\left(\frac{\langle S_y\rangle_{\pm 1/2}(t)}{\langle S_x\rangle_{\pm 1/2}(t)}\right) = \tan^{-1}\left(\frac{\pm\sin\delta\cos\theta}{\cos\delta \pm \cos\phi\sin\theta}\right), \quad (6)$$

where the index refers to the states $\zeta^{\theta,\phi}_{\pm 1/2}$ of system I. Obviously, this phase can assume values up to π for appropriate values of θ and ϕ. Since the usual evolution of a spin 1/2 under the heteronuclear J coupling leads to a phase $\delta = \pi J t$, the phase can be considered as a clock and the experimenal scheme described above generates a phase that would correspond to a different evolution period. The largest effect is obtsined for $\phi = 0$, i.e. when the I system is quantized along a direction in the xz plane. In this case, the resulting phase is

$$\delta'_{\pm 1/2}(t) = \tan^{-1}\left(\frac{\pm\sin\delta\cos\theta}{\cos\delta \pm \sin\theta}\right). \quad (7)$$

Depending on the eigenvalue chosen, the effect on the system is an accelerated (for projection onto $\zeta^{\theta,\phi}_{-1/2}$) or a retarded (projection onto $\zeta^{\theta,\phi}_{1/2}$) evolution. For our demonstration of the time-translation effect, we choose the negative eigenvalue.

The experimental realization of this scheme was performed on a ^{13}C–^{1}H spin pair of chloroform (99% ^{13}C enriched) using a Bruker MSL-400 NMR spectrometer. The coupling between the two spins due to the high-field truncated scalar coupling is then given by equation (1). The pulse sequence used is shown schematically in figure 1: the pulses which are applied to the I spin are shown in the upper trace, those which act on the S spin on the lower trace. A symbol of the form $(\kappa)_x$ denotes a radio

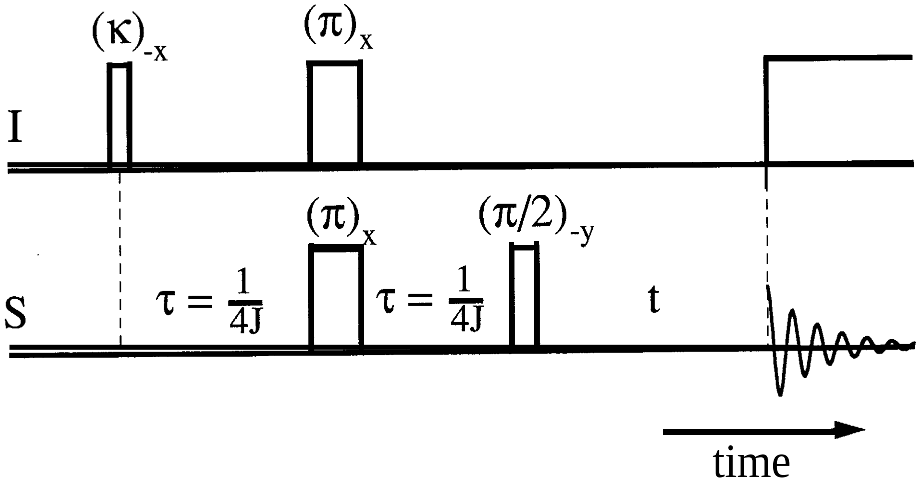

Figure 1. Pulse sequence used in the experiment. The upper line shows the pulses applied to the Z(^{1}H) spin, the lower part those applied to the S(^{13}C) spin. The flip angles of the pulses are given in parentheses, the relative phase as an index.

frequency (r.f.) pulse with flip angle κ around the x axis. The first four pulses (two on each channel) are used to prepare the initial condition equivalent to the one given in equation (2). The system was then allowed to evolve freely for a time t under the influence of the heteronuclear coupling. The rotation of the quantization axis of the I system was performed by applying an r.f. field to the ^{1}H spins along a direction in the xz plane. Simultaneously, the signal from the ^{13}C spins was recorded. By this procedure, the signal components coresponding to both states of system I are recorded simultaneously. A separation can easily be achieved via Fourier transformation of the measured free induction decay: due to the heteronuclear coupling, the two components precess at different frequencies.

Two examples of the resulting spectra are shown in figure 2. They were obtained by Fourier transformation of the free induction decay with (lower) and without (upper) an r.f. field applied to the protons. The two resonance lines correspond to the expectation values $\langle S_x \rangle_{1/2}$ and $\langle S_x \rangle_{-1/2}$ associated with the $\pm 1/2$ states of the proton (i.e. the I spin); selection of one resonance line implements therefore the projection onto a particular eigenvalue of $I_{z'}$ (see equations (4) and (5)). The asymmetric shape of the two resonance lines is caused by the precession under the heteronuclear coupling and the degree of asymmetry is a direct measure of the phase acquired by the S spins. In the upper trace, this precession angle δ is 36° and in the lower trace, the precession angle δ' of the high frequency line increases to 84°. This precession angle was reached after the same evolution period as in the upper trace, thereby demonstrating a time-translation effect.

This experiment was repeated for different tilt angles of the quantization axis. The resulting spectra are compared with the theoretically predicted spectra in figure 3. The agreement between theoretical and experimental spectra is excellent; the

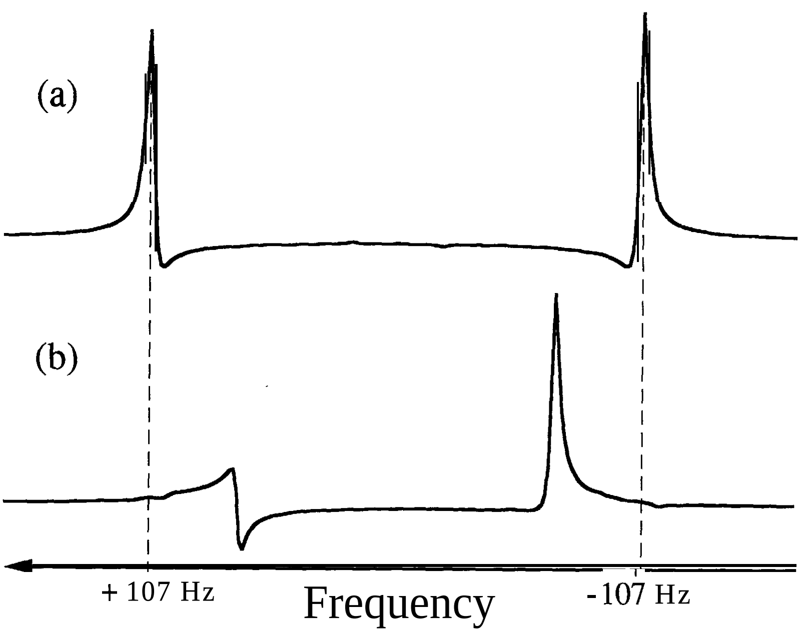

Figure 2. Spectra obtained by Fourier transformation of a free induction decay without (a) and with (b) irradiation of the I-spin system. The two resonance lines correspond to different states $m_I = \pm 1/2$ of the 1 system.

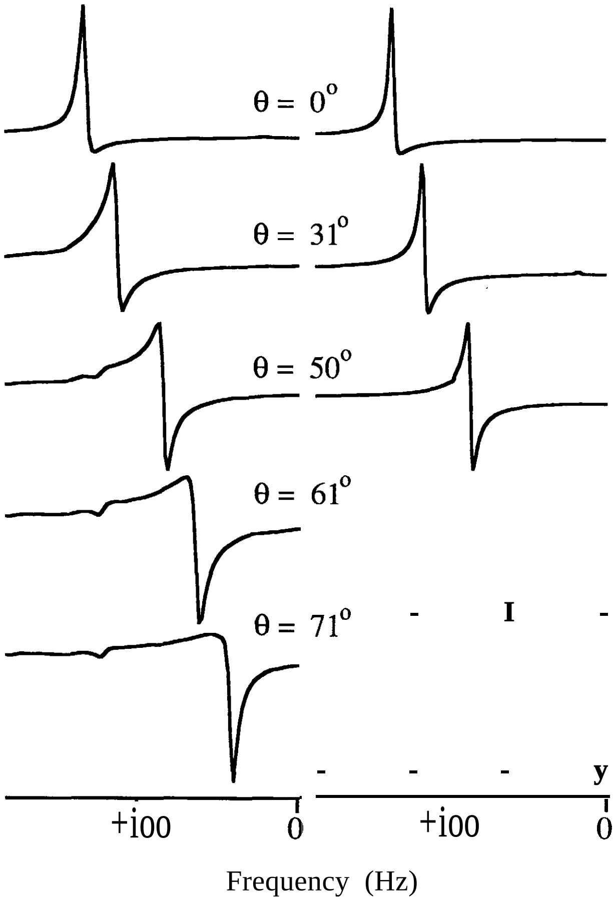

Figure 3. Comparison of experimental (left) and theoretical (right) spectra of the high frequency line of the ^{13}C spectrum for different angles between the quantization axis and the z axis.

differences are due mainly to the deviation of the experimental lineshape from the theoretically expected Lorentzian shape; the experimental lineshape is determined by the inhomogeneity of the static magnetic field and, to some extent, by the distribution of tilt angles of the quantization axis due to r.f. field inhomogeneity. The amplitudes of the resonance lines are not to scale; as seen also in figure 2, the amplitudes decrease as the phase 1 ‘amplification factor’ is increased when the quantization axis is tilted away from the z axis.

Figure 4 shows another comparison between the experimental and theoretical behaviour. For this figure, the precession angle δ' was extracted from the experimental data by a least-squares fitting procedure. The experimetnal data (circles) are compared with the theoretical curve calculated from equation (7). Since the experimental phase always tends towards π for tilt angles near $\pi/2$, irrespective of the evolution time, arbitrarily large amplification factors are possible. In practice, limitations arise. As pointed out also in reference [1], the signal amplitude decreases dramatically as the amplification factor is increased. In the exmaple shown here, the decrease in the amplitude is approximately one order of magnitude for a phase multiplier of about 4. Larger multipliers lead to correspondingly higher attenuation.

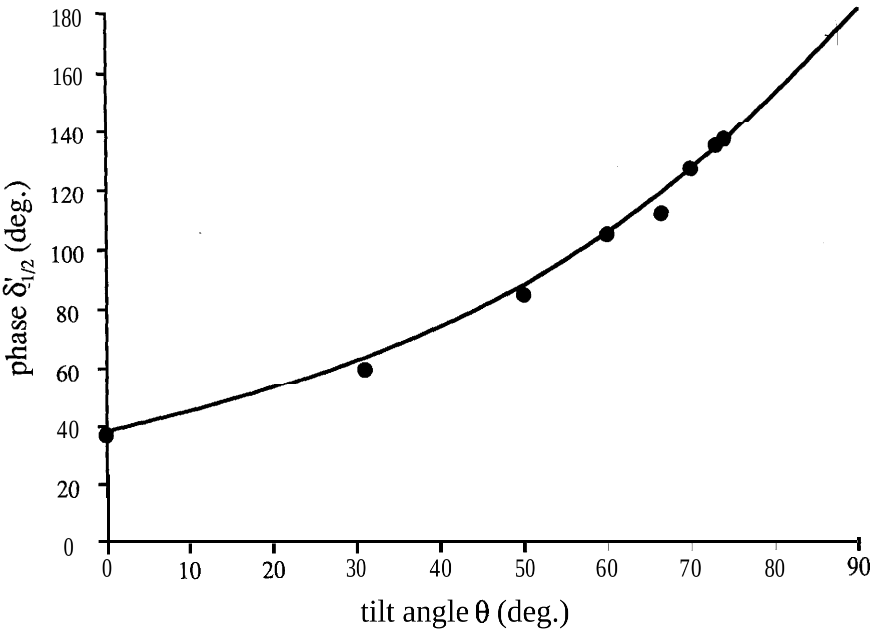

Figure 4. Experimental (circles) and theoretical (line) signal phase as a function of the tilt angle θ of the quantization axis. The experimental uncertainties are of the order of the symbol size.

While the time-translation effect seems mysterious at first, this example shows that it is fully within the realm of quantum mechanics. This can be shown more clearly by using a density operator formalism for the description of the experiment, thereby performing the average over the spin ensemble explicitly. For this purpose, we expand the density operator in terms of spin operators [2]. Starting from a thermal equilibrium density operator at high temperature $\rho_{eq} = \frac{1}{4}\mathbb{1} + \hbar\omega_S/kT(S_z + 4I_z)$ with ω_S representing the Larmor frequency of spin S, the first four pulses prepare the system in the state

$$\rho(0) = \tfrac{1}{4}\mathbb{1} + \frac{\hbar\omega_s}{kT}(S_x + 8\sin\kappa S_x I_x - 4\cos\kappa\, I_z), \tag{8}$$

where κ represents the flip angle of the first pulse. A state equivalent to $\Psi_{SI}(0)$ of equation (2) can therefore be obtained by setting $\sin\kappa = 0{\cdot}25$: apart from irrelevant terms proportional to I_x and I_z, and neglecting the Boltzmann factor, the simplified initial density operator σ (0) is then

$$\sigma(0) = |\Psi_{SI}(0)\rangle\langle\Psi_{SI}(0)| = \tfrac{1}{4}\mathbb{1} + \tfrac{1}{2}I_x + \tfrac{1}{2}S_x + S_x I_x, \tag{9}$$

the density operator equivalent of equation (2). The effect of the free precession under the heteronuclear coupling is described by the unitary transformation $U_1 = \exp(-i\delta 2 S_z I_z)$ with $\delta = \pi Jt$ which leads to the state

$$\begin{aligned}\sigma(1) &= U_1\sigma(0)U_1^{-1}\\ &= \tfrac{1}{4}\mathbb{1} + \tfrac{1}{2}I_x\cos\delta + S_z I_y\sin\delta + \tfrac{1}{2}S_x\cos\delta + S_y I_z\sin\delta + S_x I_x.\end{aligned} \tag{10}$$

We turn now to the special case where the quantization axis for the I spin system lies in the xz plane at an angle θ with respect to the old z axis. The transformation to this coordinate system is achieved by the rotation $U_2 = \exp(-i\theta I_y)$ so that

$$\begin{aligned}\sigma(2) &= U_2^{-1}\sigma(1)U_2 \\ &= \tfrac{1}{4}\mathbb{1} + \tfrac{1}{2}I_x\cos\delta\cos\theta + \tfrac{1}{2}I_z\cos\delta\sin\theta + S_zI_y\sin\delta + \tfrac{1}{2}S_x\cos\delta \\ &\quad + S_yI_z\sin\delta\cos\theta - S_yI_x\sin\delta\sin\theta + S_xI_x\cos\theta + S_xI_z\sin\theta. \end{aligned} \tag{11}$$

The measured quantities are then easily calculated as

$$\begin{aligned}\tfrac{1}{2}\langle S_x \pm 2S_xI_z\rangle &= \tfrac{1}{2}\mathrm{Tr}\{\sigma(2)\,(S_x \pm 2S_xI_z)\} = \tfrac{1}{4}(\cos\delta \pm \sin\theta) \\ \tfrac{1}{2}\langle S_y \pm 2S_yI_z\rangle &= \tfrac{1}{2}\mathrm{Tr}\{\sigma(2)\,(S_y \pm 2S_yI_z)\} = \pm\tfrac{1}{4}\sin\delta\cos\theta, \end{aligned} \tag{12}$$

So that the observed phase becomes

$$\delta'_{\pm 1/2}(t) = \tan^{-1}\frac{\langle S_x \pm 2S_xI_z\rangle(t)}{\langle S_y \pm 2S_yI_z\rangle(t)} = \tan^{-1}\left(\frac{\pm\sin\delta\cos\theta}{\cos\delta \pm \sin\theta}\right), \tag{13}$$

in agreement with equation (7). This form shows clearly that the augmented phase is due to the competition of the two terms in the denominator, which allows one to make the denominator arbitrarily small.

The calculation also suggests a different way of performing the experiment: instead of applying a constant r.f. field at an angle θ from the z axis during detection, the same result could be obtained by applying a pulse with flip angle θ along the y direction immediately before detection. In this case, the resonance lines would be observed at their native frequencies but the phases and amplitudes would be modified as in the current experiment. The near cancellation of two terms in the denominator of equation (13) shows that the time-translation effect is related to the possibility of measuring values far outside of the eigenvalue spectrum of the observable [3] which is claimed to be possible in certain 'weak measurements'[4].

The density operator formalism presented here also suggests an alternative interpretation of the observed effect that does not require the time translation concept; the entangled state $\Psi_{SI}(0)$ does not allow independent measurements on both spins. Selecting a particular eigenvalue of the I spin selects therefore also among the S spins; excluding some of the S spins from the actual measurement process leads to the reduced amplitude discussed above. This interpretation permits also the construction of a classical analogue that does not involve a superposition of time evolutions: suppose that an ensemble of atoms whose velocities obey a Boltzmann distribution is initially located in the half space $x < 0$ and that a shutter at $x = 0$ is suddenly removed, allowing the atoms to spread into the other half space. If the progress of the atoms into the right half space is monitored by irradiating the atoms with a laser beam incident from $x \to \infty$ and measuring the fluorescence of the atoms, the result clearly depends on the laser frequency which selects a certain velocity group within the Doppler-broadened resonance line. If the laser frequency is set to the red wing of the resonance line, only the atoms with a high velocity component in the x direction are excited, so that the motion of the atoms appears faster than would be expected from the thermal velocity distribution. A negative scaling factor could equally be obtained by tuning the laser to the blue side of the spectrum, thereby choosing the atoms with a velocity component in the $(-x)$ direction, i.e. the atoms moving away from the shutter region into the filled half-space. Neglecting the effect of collisions, it would then appear as if the half-space to the right remained empty.

In conclusion, we have shown that the time-translation machine suggested by Aharonov *et al.* can actually be realized experimentally. For this implementation, we have used a spin system coupled to a quantum variable that is in our case a second spin variable. Similar experiments with classical systems seem equally possible. The approach used here leads to a time translation only if the system is initially in a specific state, as in all the examples given in reference [l]. No simple generalization is apparent that would lead to the more general case of a time-translation that is independent of the initial conditions, as mentioned in reference [l].

While our experiment can be considered as an implementation of the time-translation concept of Aharonov *et al.*, this is certainly not the only possible interpretation, as we have shown. We hope that our explicit treatment has somewhat demystified the 'time-translation machine'. In the context of the described simple experiment, its function is a straightforward consequence of well known quantum mechanical measurement principles. Nevertheless, we do not want to preclude that the principles presented in reference [1] could have more far reaching consequences that go beyond our present understanding.

This work was supported by the Schweizerischer Nationalfonds. We gratefully acknowledge helpful discussions with Dr Rafael Brüschweiler.

References

[1] AHARONOV, Y., ANANDAN, J., POPESCU, S., and VAIDMAN, L., 1990, *Phys. Rev. Lett.*, **64**, 2965.

[2] ERNST, R. R., BODENHAUSEN, G., and WOKAUN, A., 1987, ***Principles of Nuclear Magnetic Resonance in One and Two Dimensions*** (Clarendon Press).

[3] AHARONOV, Y., ALBERT, D. Z., and VAIDMAN, L., 1988, *Phys. Rev. Lett.*, **60**, 1351.

[4] DUCK, I. M., STEVENSON, P. M., and SUDARSHAN, E. C. G., 1989, *Phys. Rev.* D, **40**, 2112.